Coherence

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1) Description of light in the phase space

First-order spatial coherence: Experiments

First –order temporal coherence

Description of light in the 6-dimensional phase space

2) Characteristics of undulator radiation

3) Second-order coherence and photon statistics

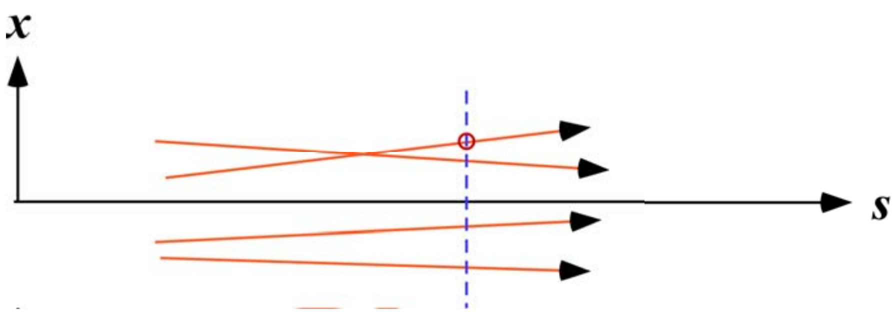
Experiments: two-photon correlation

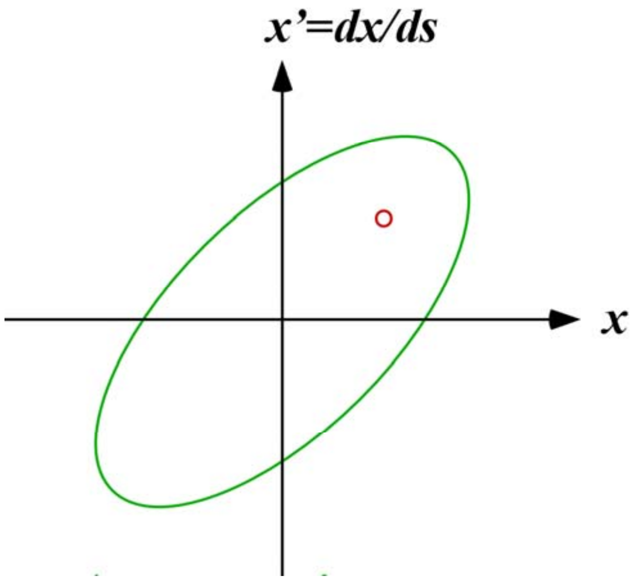
4) Coherence and density matrix

Observation of subspace, decoherence

5) Outlook

Ⅰ. Description of light in the $(x, x', y, y', \omega, t)$ space

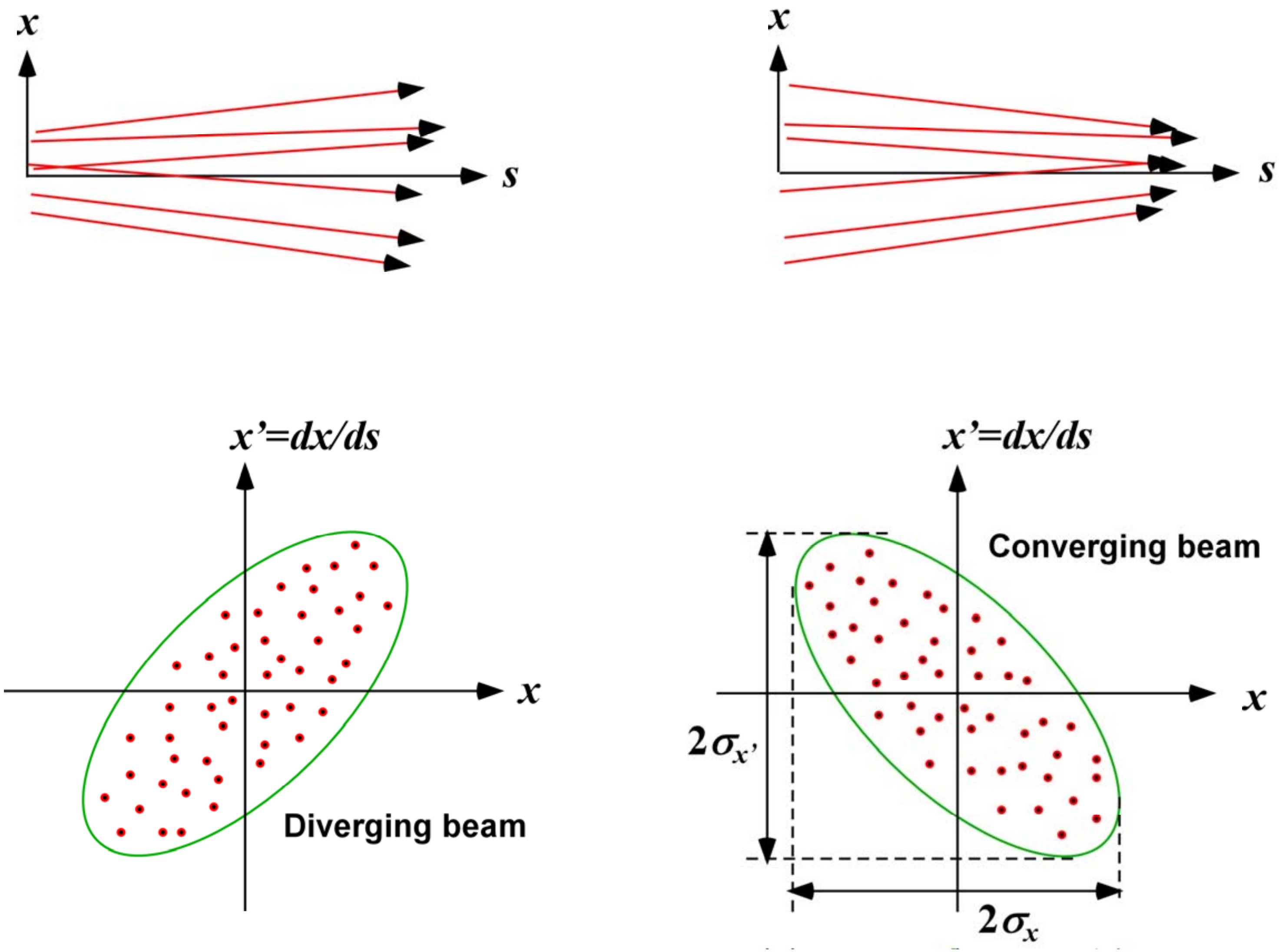




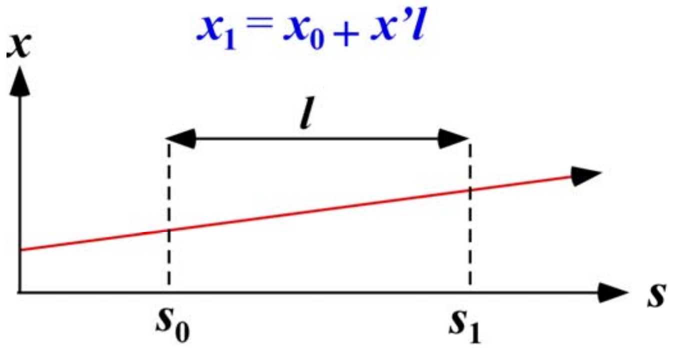
Trick:

Describe light geometrically and introduce uncertainty principle of light (Fourier limit)

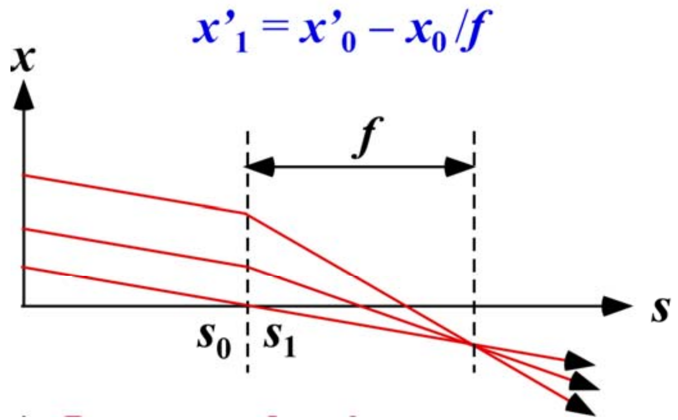
ω–t space is treated as same as the position – momentum space



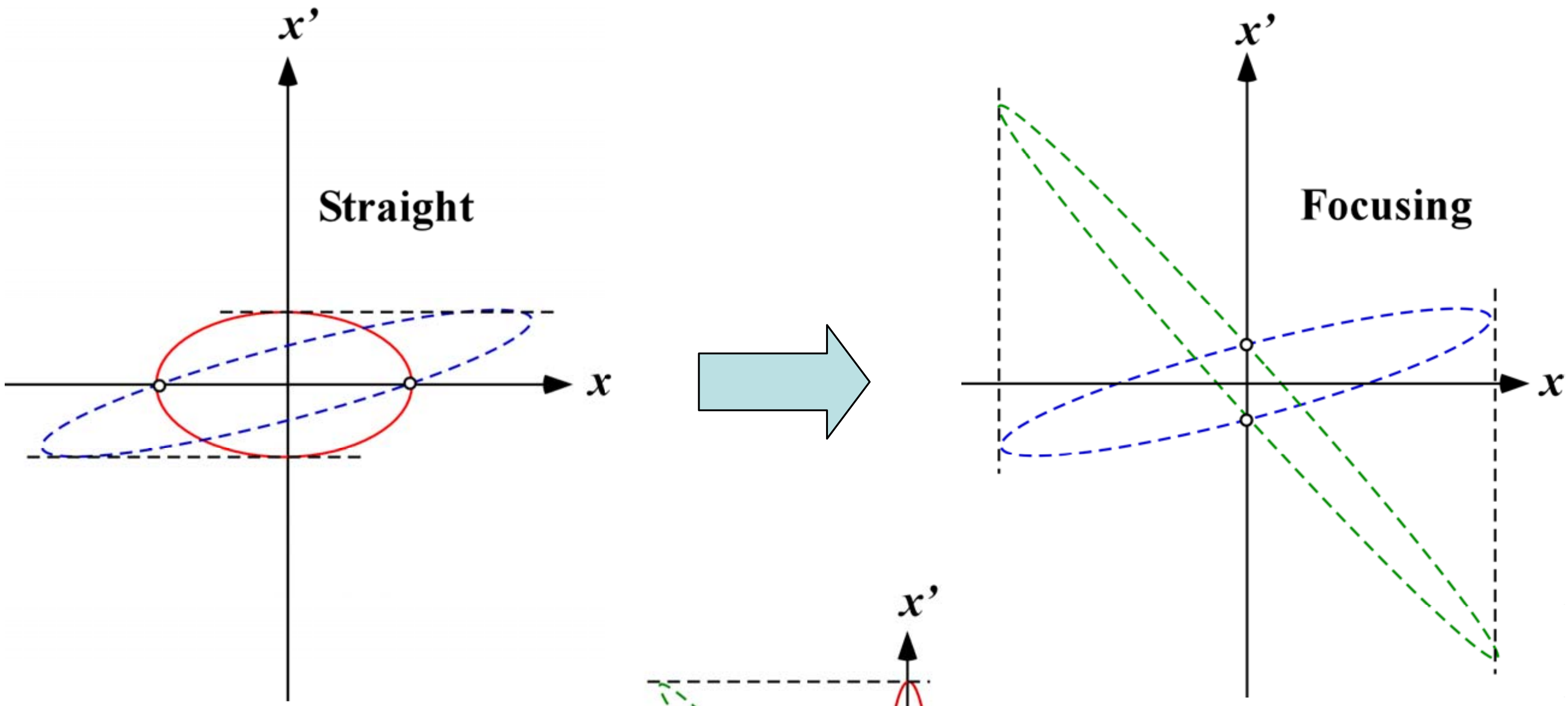
x
s
x
s
x'=dx/ds
x
Diverging beam
x'=dx/ds
Converging beam
x
$2\sigma_{x'}$
$2\sigma_x$

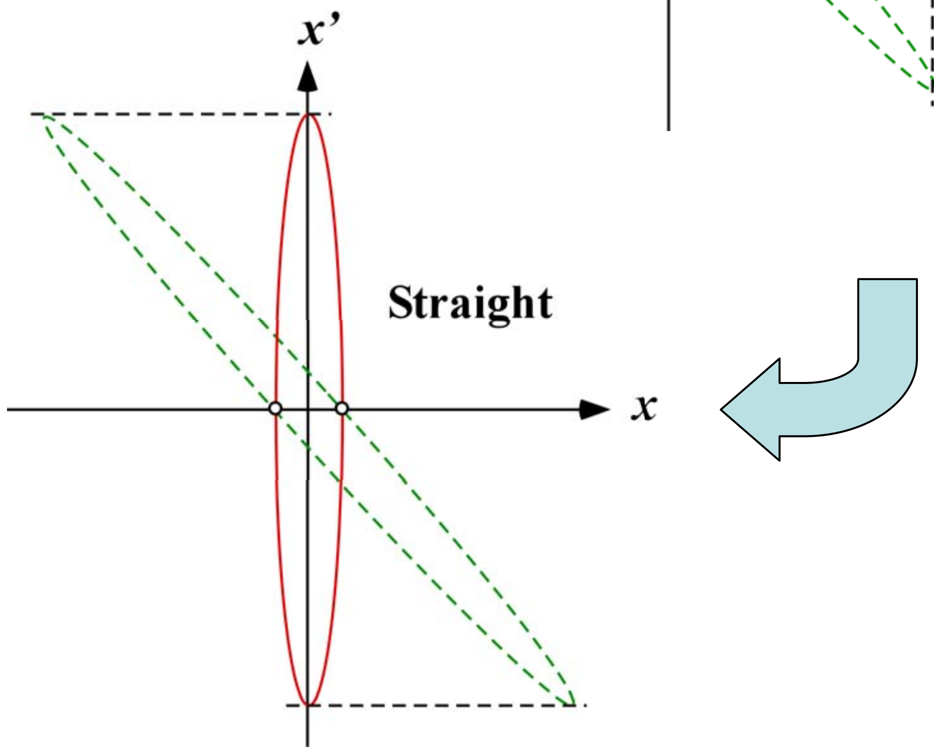


$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$



$$\begin{pmatrix} x_1 \\ x'_1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ \mp 1/f & 1 \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$

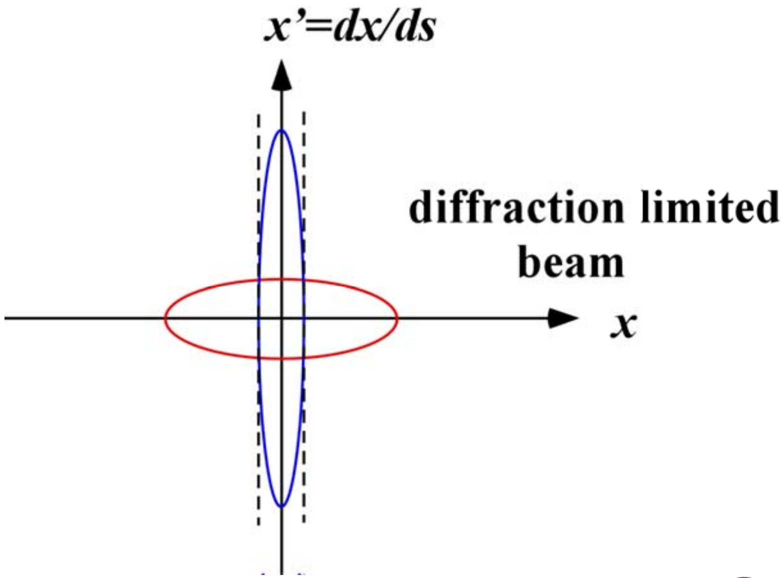




Changing beam size

Diffraction limited beam

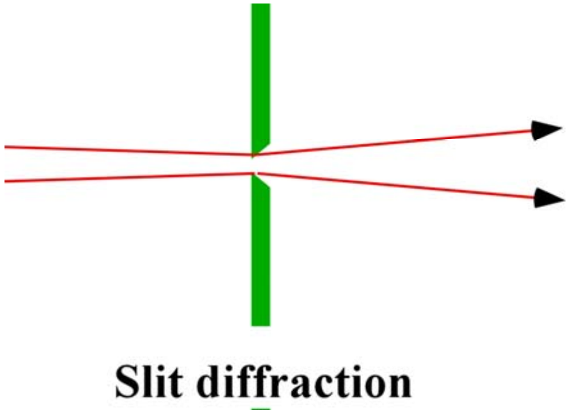
Because of uncertainty principle the minimum area of the ellipse $= \lambda/4$



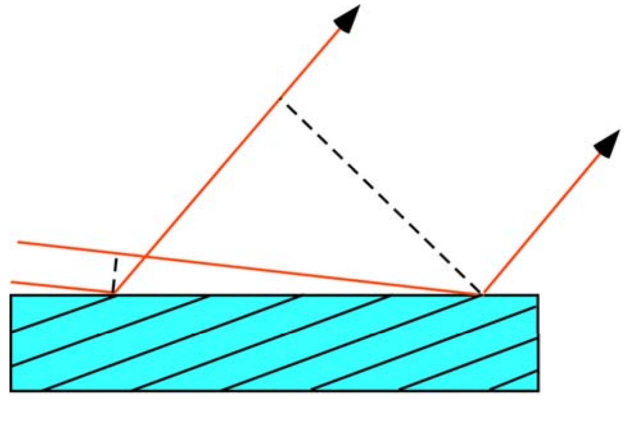
Downsizing the beam makes the beam divergence larger

Gaussian beam: Beam with standard deviation of distribution described by an ellipse

Conservation of the emittance of diffraction limitted beam

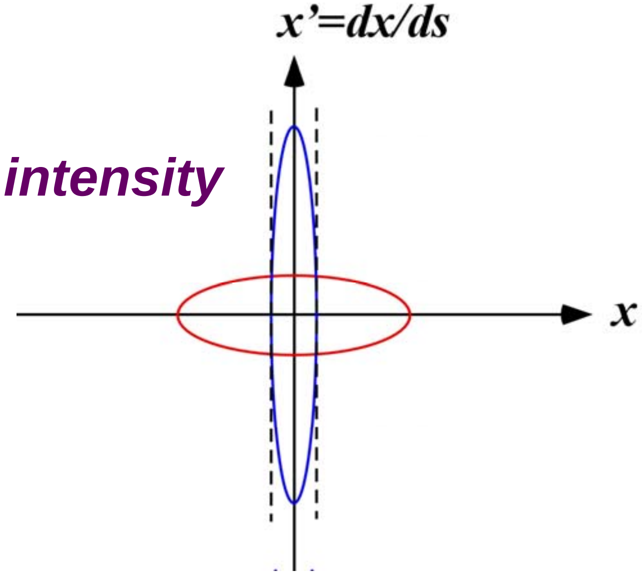


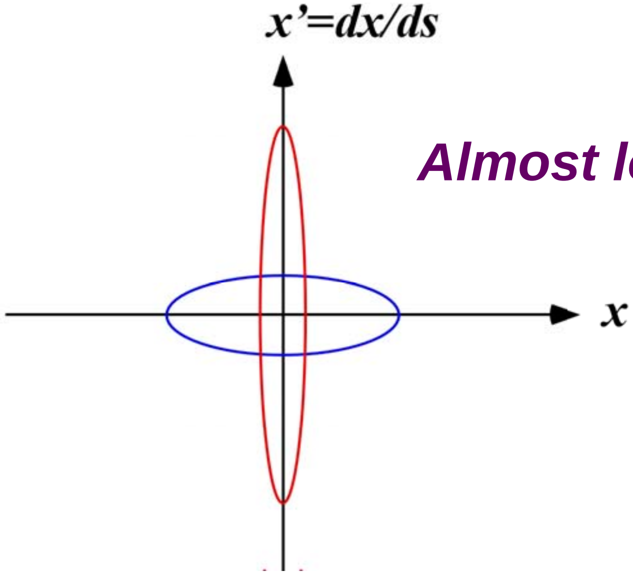
Slit diffraction



Asymmetric Bragg reflection

Loss of intensity



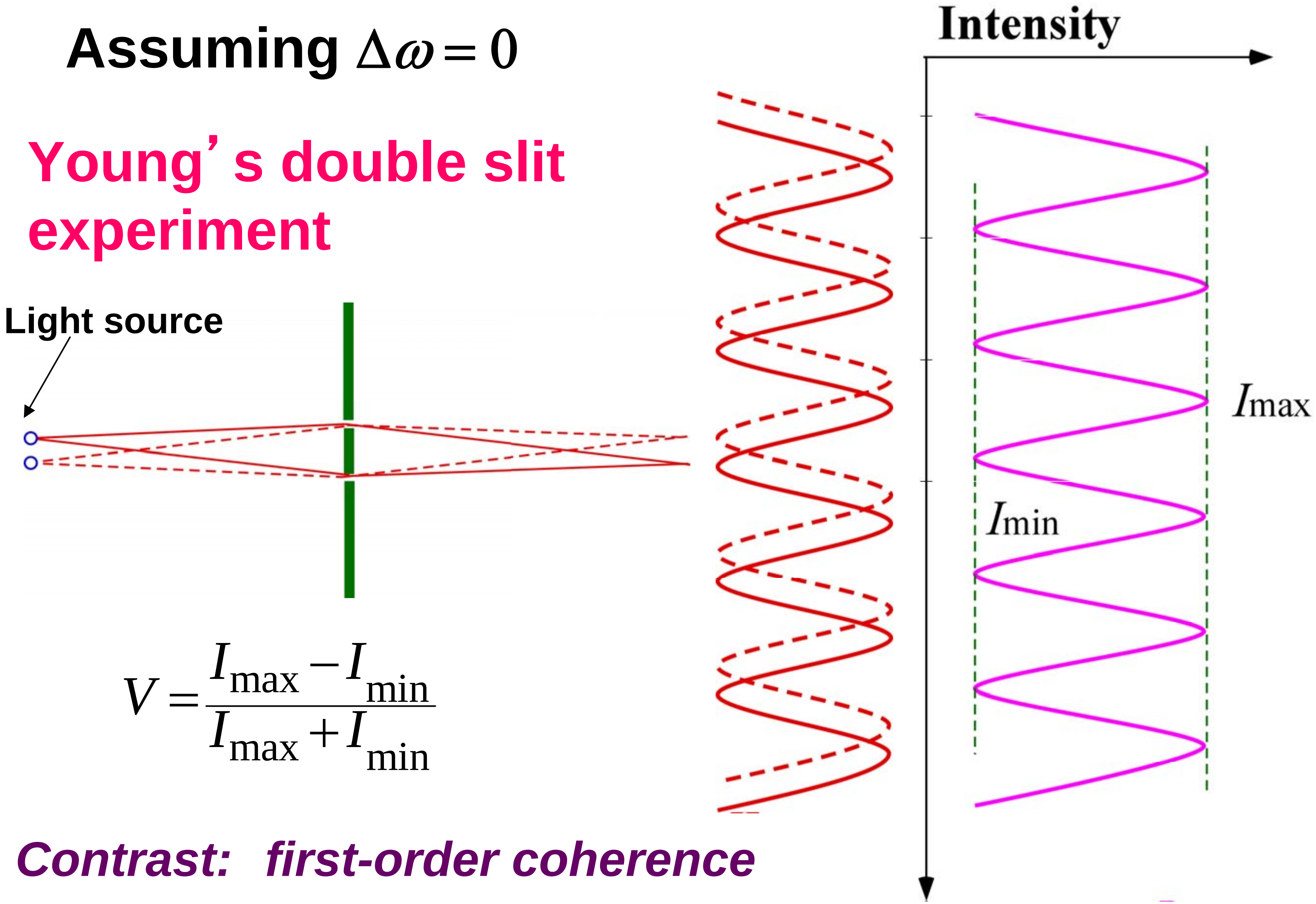


Almost lossless

First- order spatial coherence

Assuming $\Delta\omega = 0$

Young's double slit experiment



$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

Contrast: first-order coherence

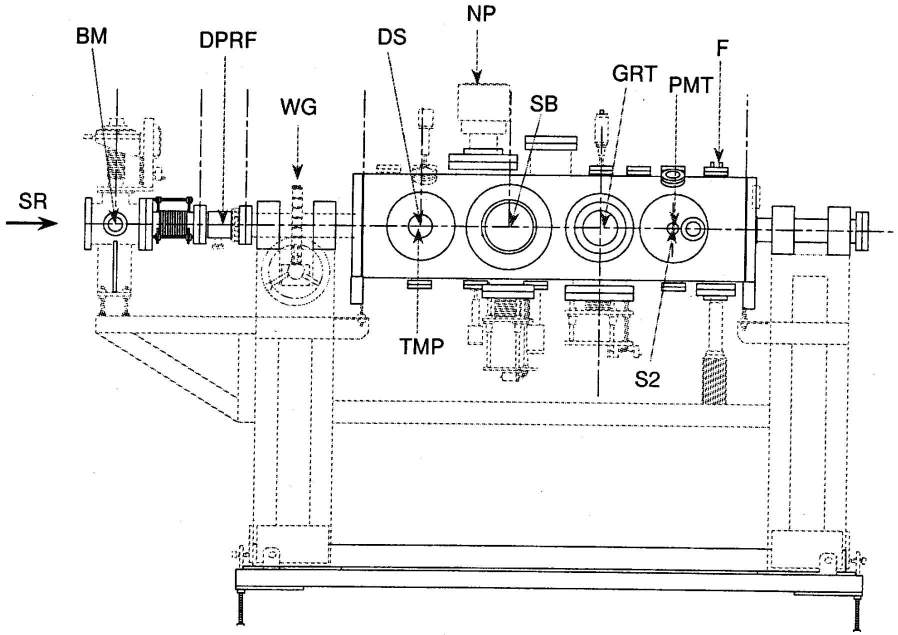


Figure 4.4: Side view of the Young's interferometer.

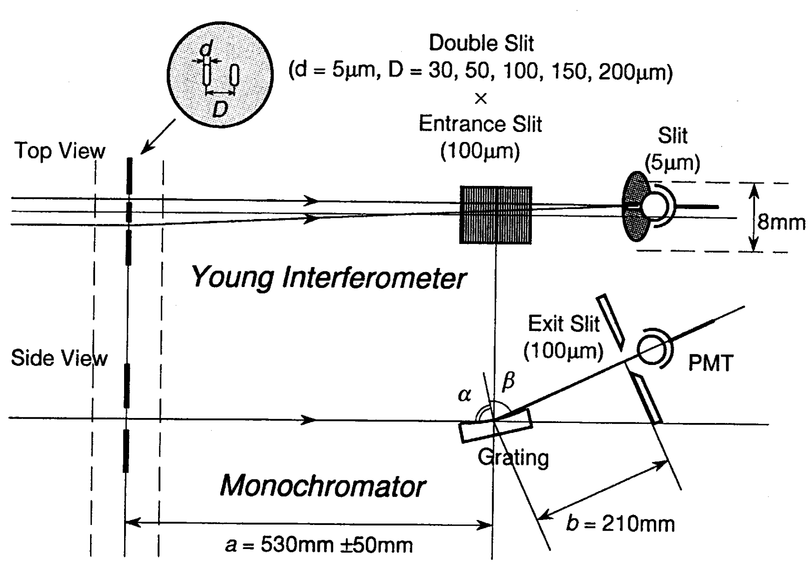


Figure 4.5: Design of the monochromator.

Y. Takayama (Doctor theses)

Undulator radiation

Bending radiation

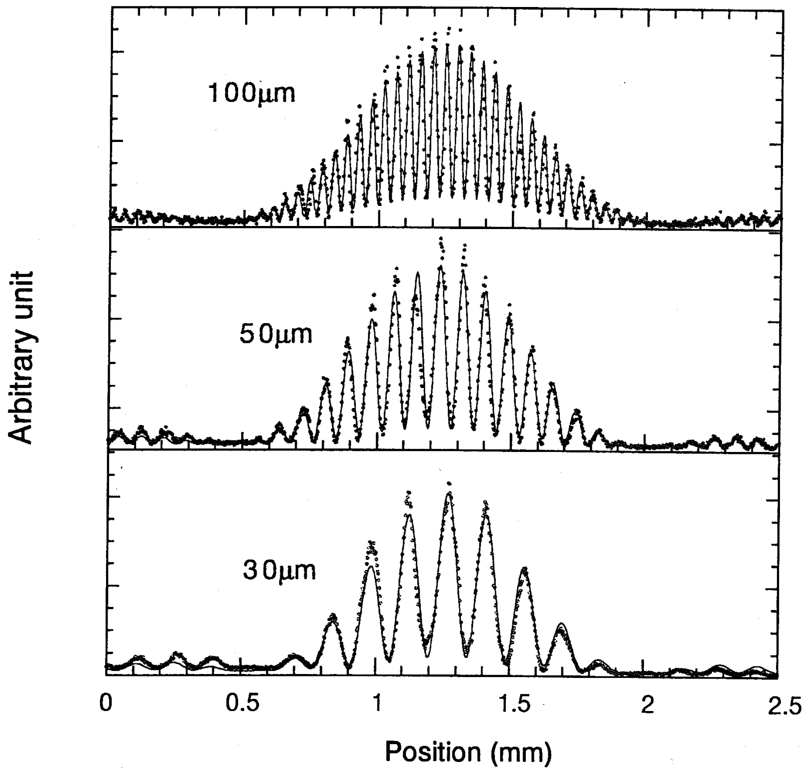


Figure 5.5: The interference patterns for $E = 100$ eV at BL-12A. The direction of the double slit is vertical.

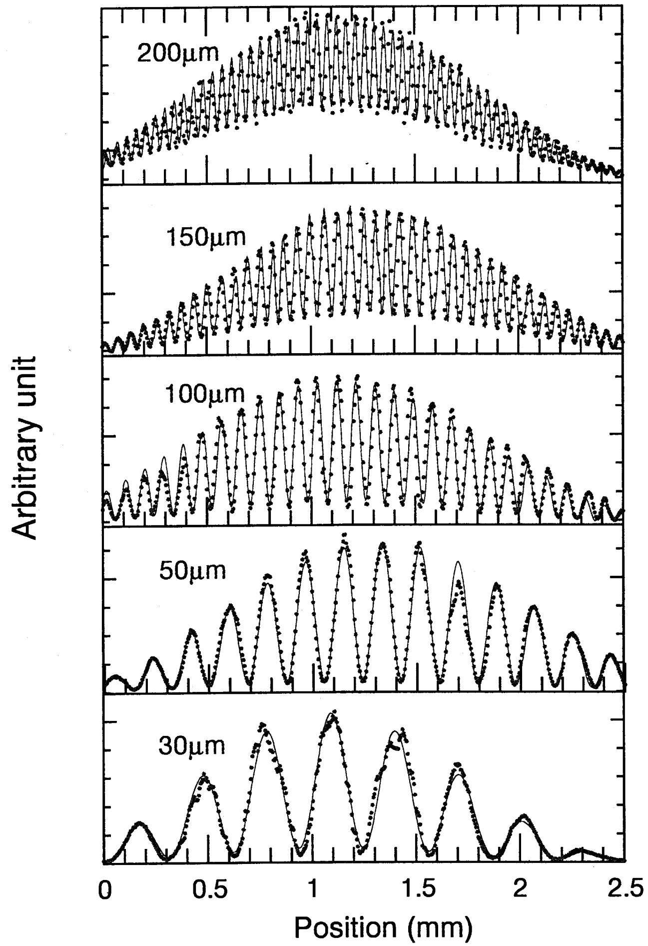
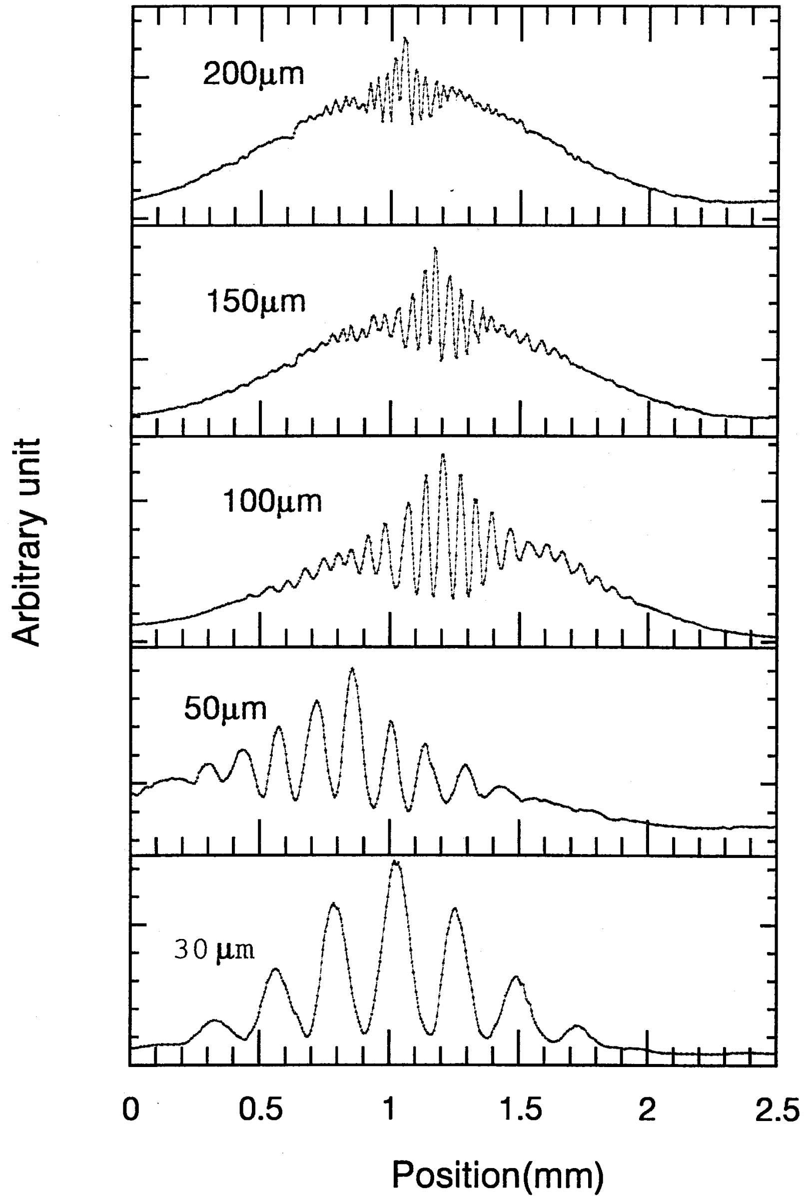
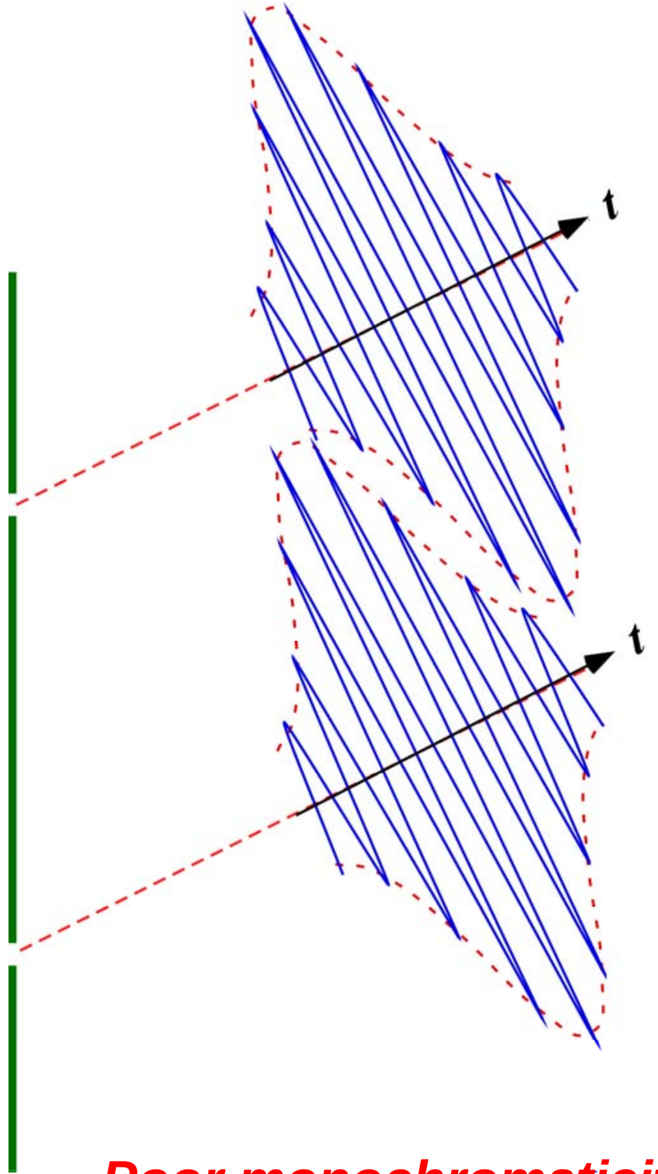


Figure 5.15: The interference patterns for $E = 100$ eV at BL-28A. The direction of the double slit is vertical.



Undulator radiation without monochromator

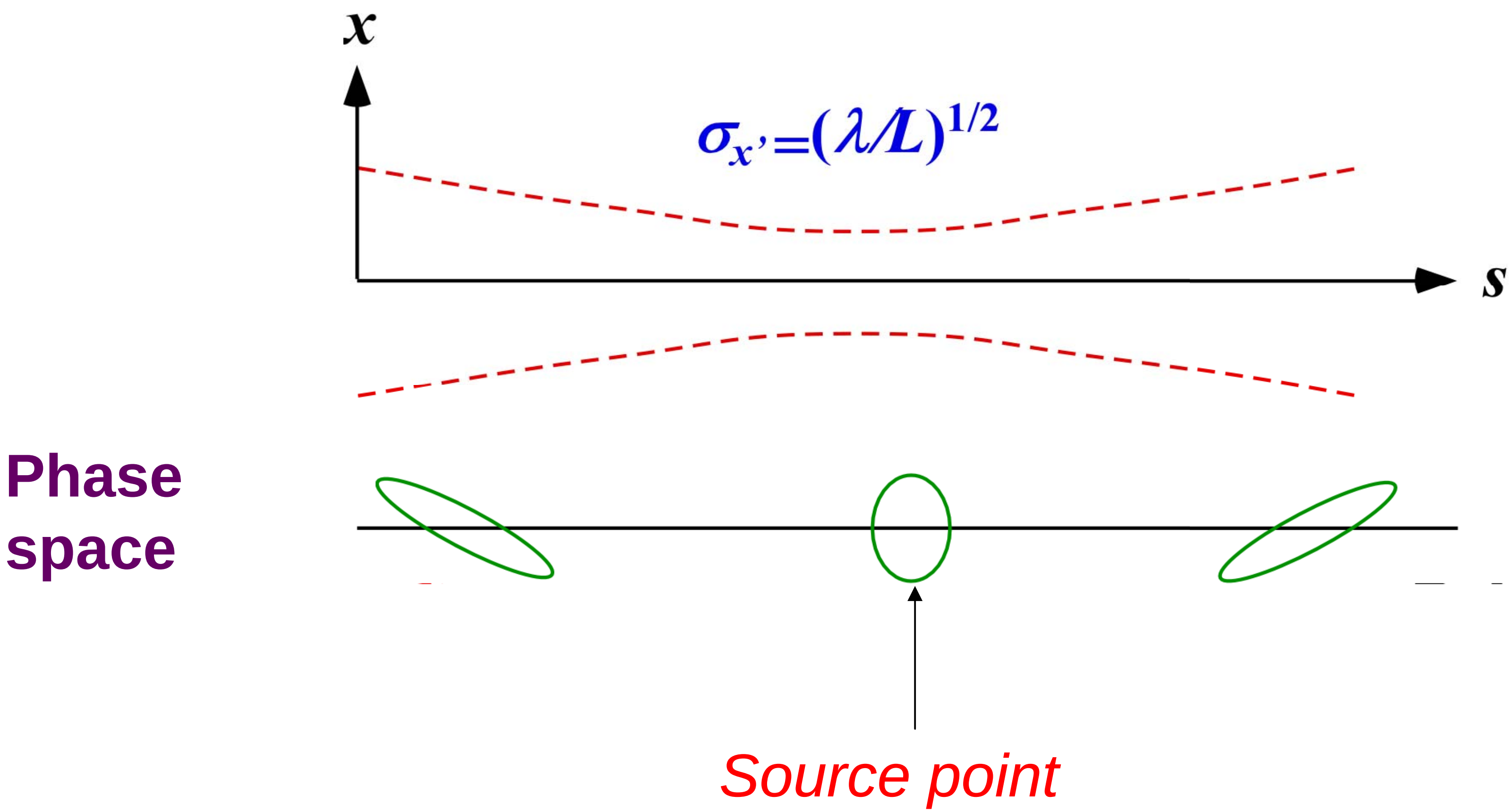


Poor monochromaticity

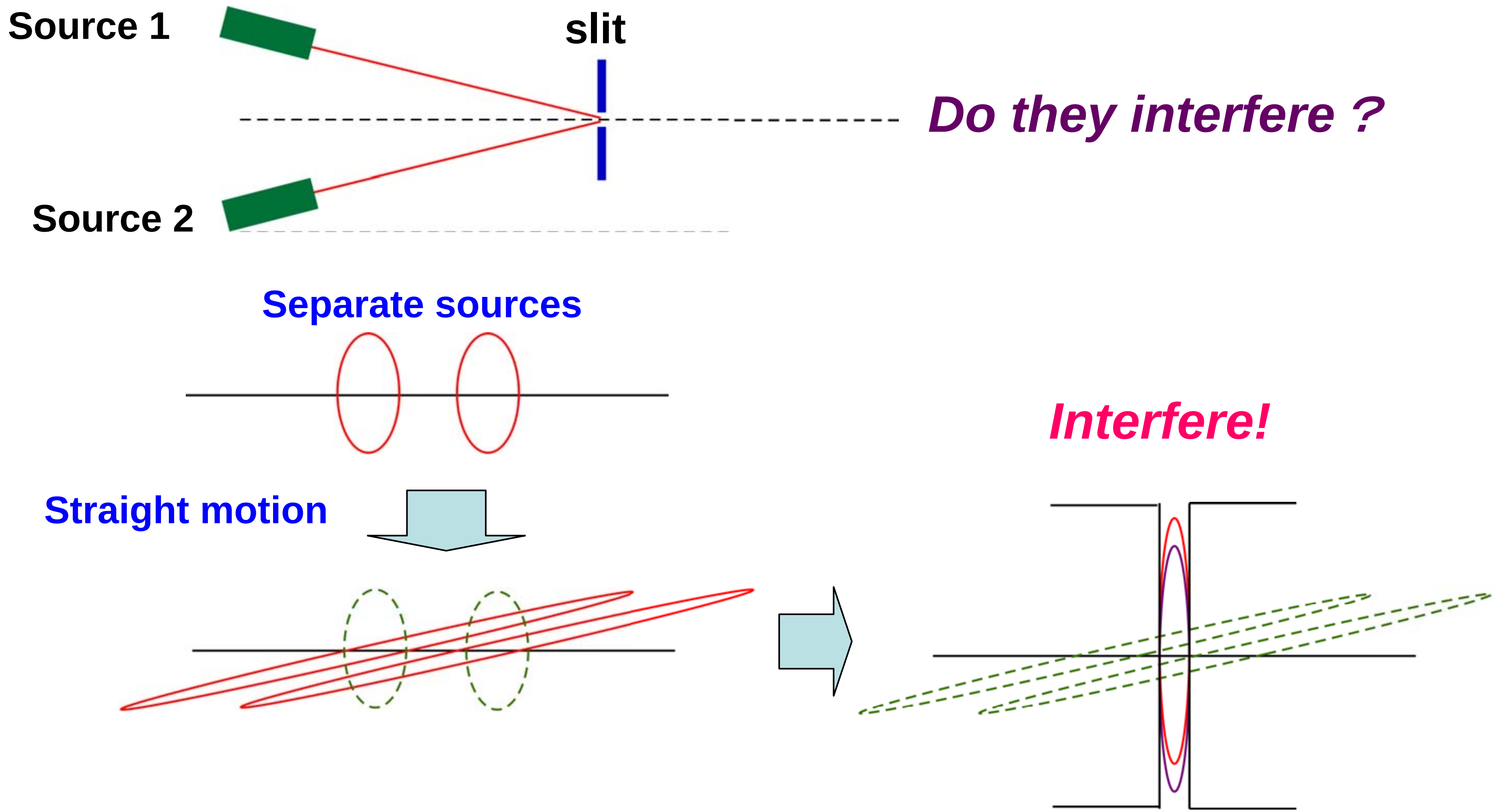
Figure 5.21: The interference patterns for the direct beam at BL-28A. The direction of the double slit is vertical.

Where is the source point of undulator radiation?

When the electron emittance is much smaller than the diffraction limit,

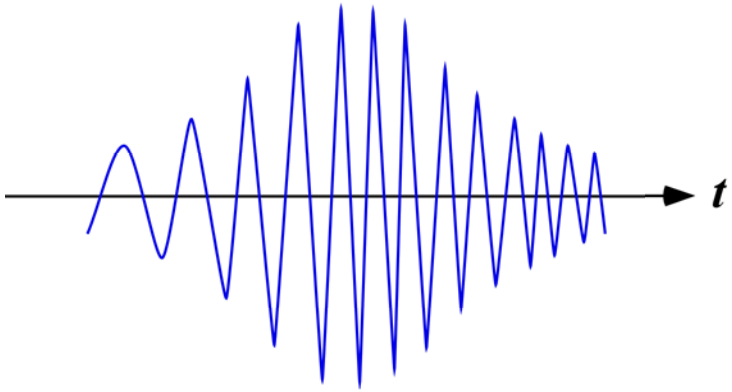


First -order coherence depends on observation

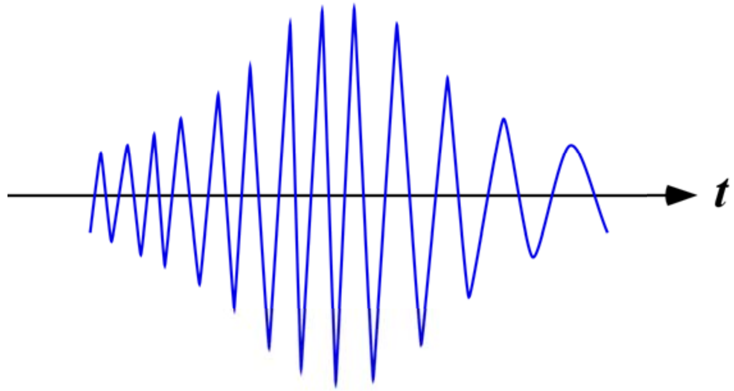


The similar thing happens in ω-t space.

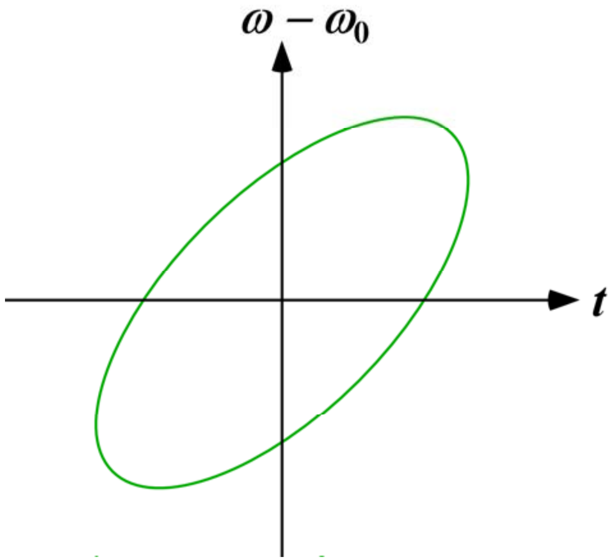
Description in ω–t space

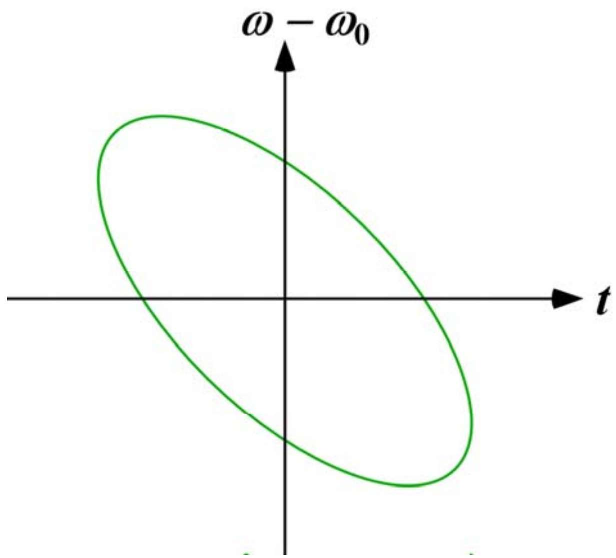


down-chirped pulse



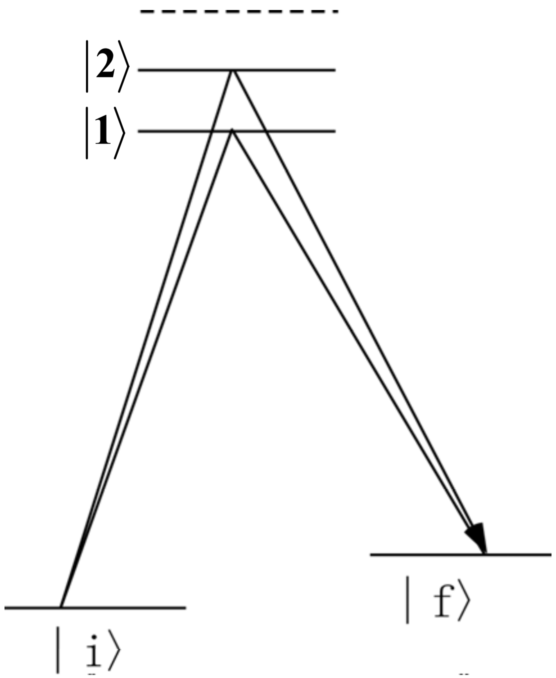
up-chirped pulse





Temporal “Young’s” interference

"Dynamical" quantum beats (more degrees of freedom)



C_1:probability to come to 1(time-dependent)

C_2:probability to come to 2(time-dependent)

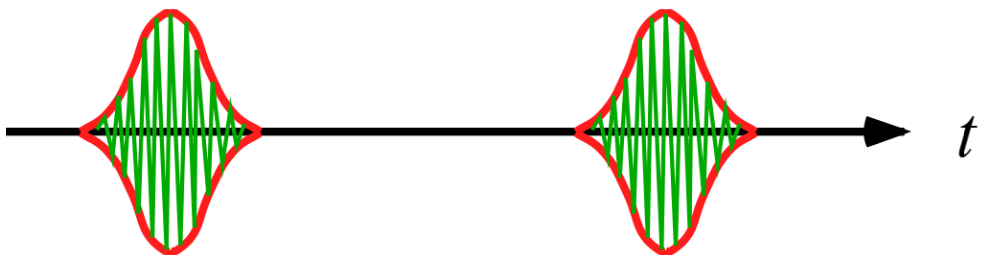
....

C_n:probability to come to n(time-dependent)

＊Special case: coherent motion

$C_n \propto (1/n!)^{1/2} \exp(-in\Omega t) A^n$

Experiments by P.Corkum *et. al*.



Phase relation between the two wave packet

Second-order coherence (Quantum mechanics)

First-order

$$\gamma_{(1)} = \frac{\langle a_2^* a_1 \rangle}{\sqrt{\langle a_1^* a_1 \rangle \langle a_2^* a_2 \rangle}}$$

correlation between *amplitudes*

Second-order

$$\gamma_{(2)} = \frac{\langle a_2^* a_1^* a_1 a_2 \rangle}{\langle a_1^* a_1 \rangle \langle a_2^* a_2 \rangle}$$

Correlation between intensities

If 1 and 2 are the same mode,

$$\gamma_{(2)} = \frac{\langle a^* a a^* a \rangle - \langle a^* a \rangle}{\langle a^* a \rangle^2}$$

Influenced by first-order coherence

Measurement of Second-order coherence

Not simple

$$S \propto I_1 I_2 \left(\tilde{A} + \kappa \frac{\tau_c}{T_R} \left| \gamma_{12} \right|^2 \right)$$

How can we eliminate the false correlation?

R. Z. Tai et al. Phys. Rev. A **60** (1999)

Two-photon correlation is proportional to wave packet length.

Width of the slit *D* is changed to change γ_{12}

A : **accidental correlation**

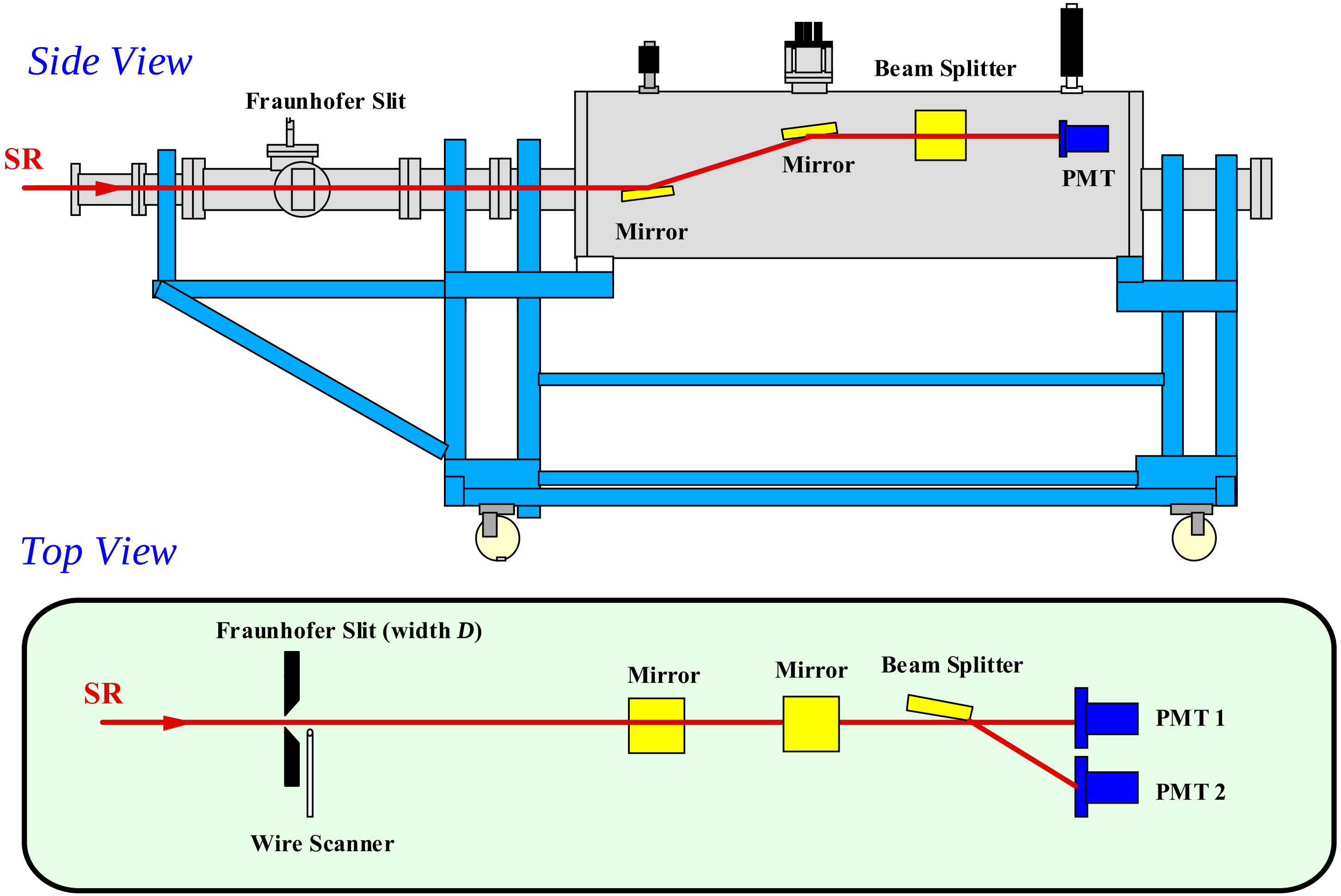
κ : **duty ratio of signals**

T_R : **response time of detectors**

τ_c : **wave packet length**

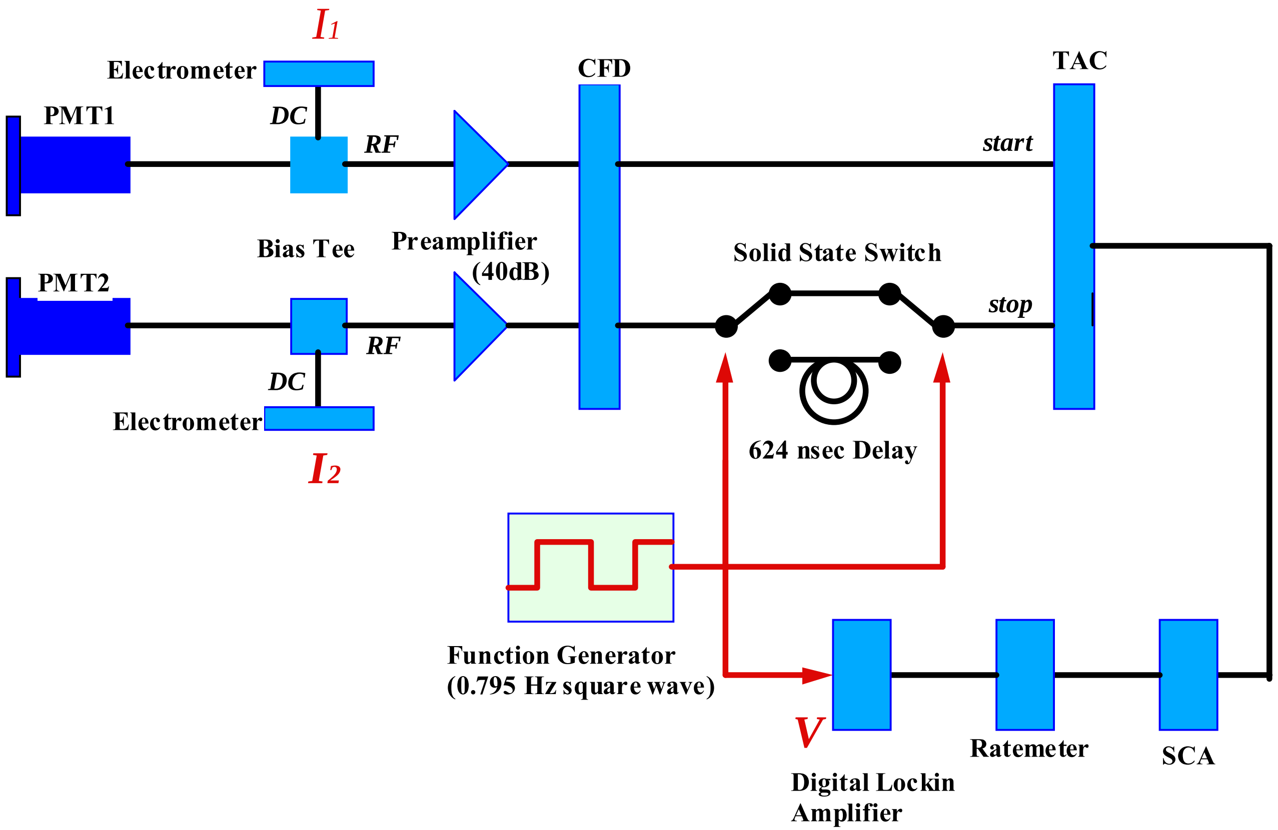
γ_{12} : **first-order spatial coherence**

Design of the Vacuum Chamber



Tai et. al., Rev. Sci. Instrum. **71** (2000) 1256.

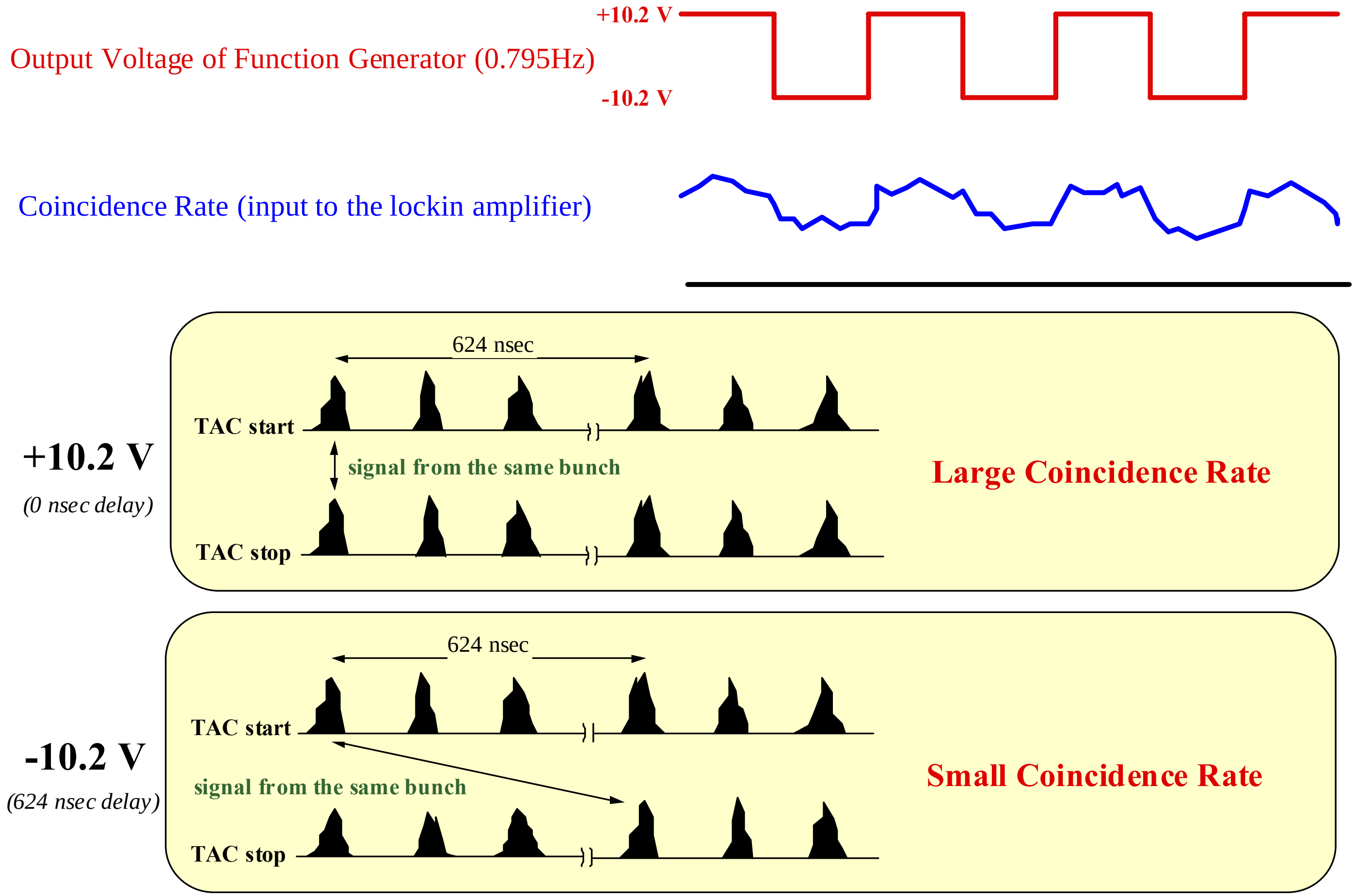
Brief Diagram of the Electric Circuit



$V_x = G(D)\, I_1\, I_2 + N_x$

G(D) is of the second order spatial coherence on the Fraunhofer slit.

Timing of Delay-Time Modulation and Control Voltage

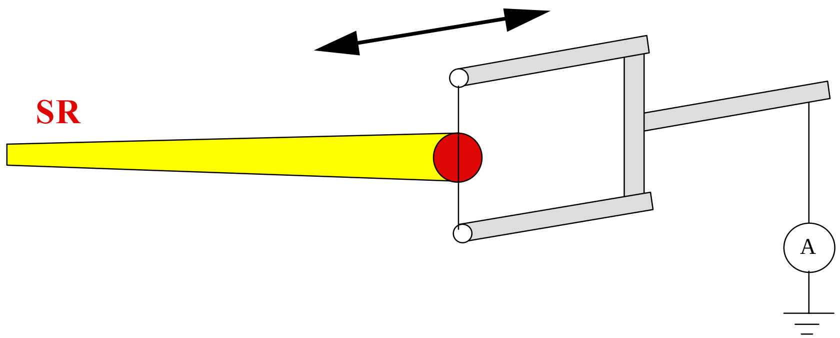


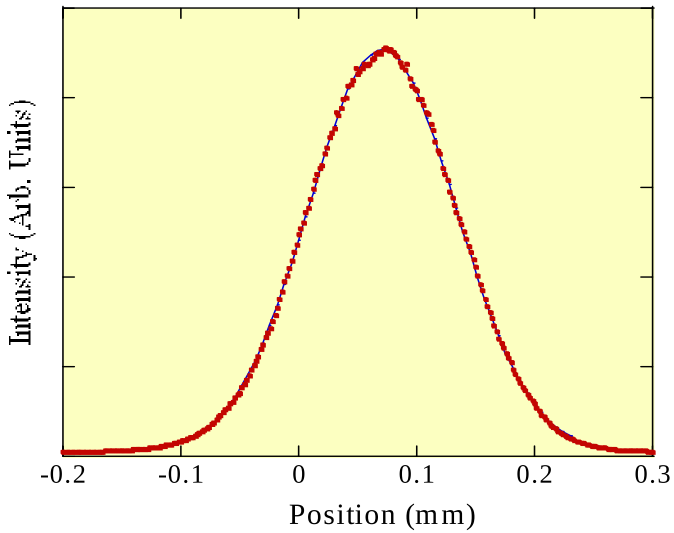
Experimental Condition

- Photon Energy 55 eV

  (energy resolution $E/\Delta E \sim 10000$)
- Coherence in the horizontal direction was measured.
- Accumulation time for the measurement of the two-photon correlation for a slit width was about 4 hours.

Beam size Measurement

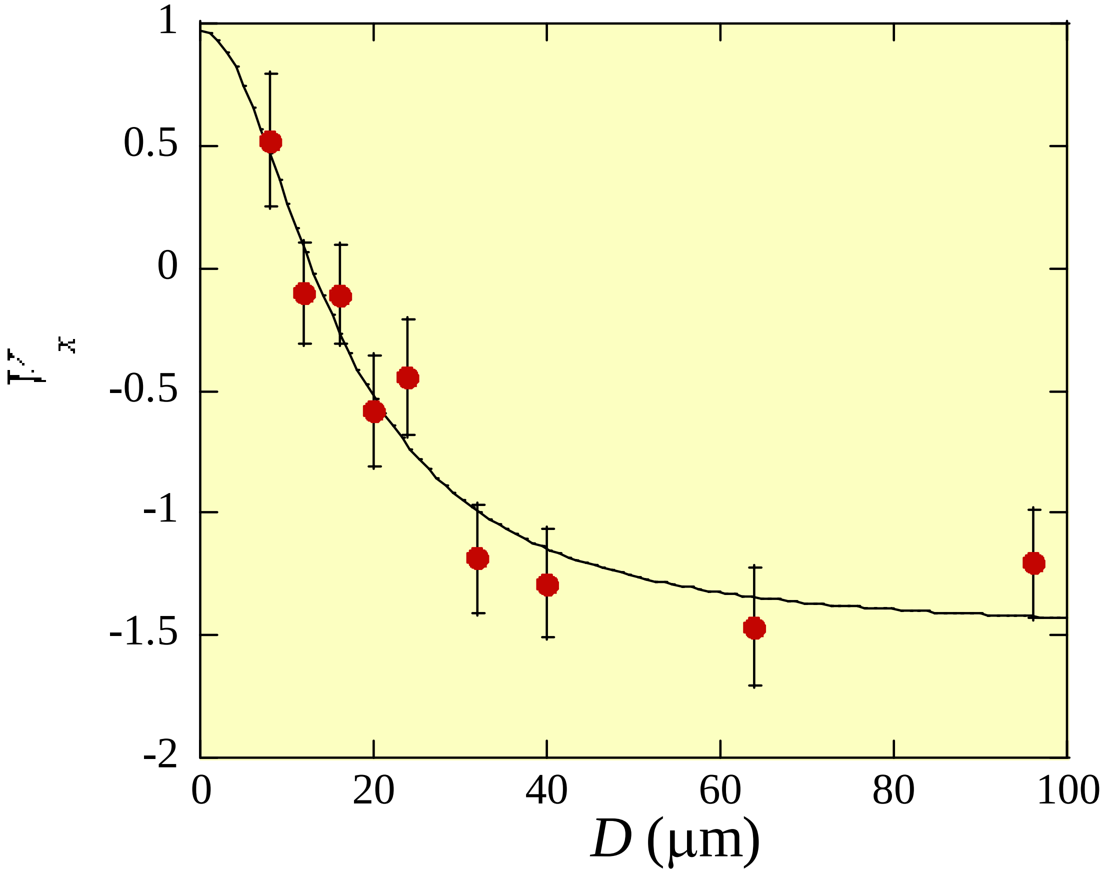




- Tungsten-wire scanner (50 μm thickness) was used.
- Beamsize $\Sigma = 60.9\ \mu\text{m}$

  (Gaussian Approximation $I(x) = I(0)\exp(-x^2/(2\,\Sigma^2))$)

An example of two-photon correlation



Characteristic of chaotic radiation

R.Z. Tai et. al., Phys. Rev. A60 3262 (1999)
Y. Takayama et al. ,J. Synchrotron Rad.10 303 (2003)

Density matrix with two spaces

subspace a, b**:** **whole space**: $|a\rangle \otimes |b\rangle$

vectors in a: $\alpha, \beta, \gamma, \delta \cdots$

vectors in b: $k, l, m, p, q \cdots$

density matrix ρ:

$$\boxed{\boldsymbol{\rho} = \sum_{\alpha\beta}\sum_{kl} |\alpha\rangle |k\rangle \langle l| \langle \beta| \rho_{\alpha\beta kl}}$$

with

$$\sum_{\alpha k} \rho_{\alpha\alpha kk} = 1$$

1) expectation value of operator A

$$\langle \mathrm{A} \rangle = \mathrm{Tr}(\boldsymbol{\rho}\,\mathrm{A}) = \sum_{\gamma m}\sum_{\beta l} \rho_{\gamma\beta ml} \langle \beta | \langle l | A | m \rangle | \gamma \rangle$$

When A does nothing on b **(not observing** b**)**

$$\langle \mathrm{A} \rangle = \sum_{l}\sum_{\beta\gamma} \rho_{\gamma\beta ll} \langle \beta | \mathrm{A} | \gamma \rangle = \sum_{k}\sum_{\alpha\beta} \rho_{\beta\alpha kk} \langle \beta | \mathrm{A} | \alpha \rangle$$

Coherence and density matrix

When the space *b* is not observed,

$$\boldsymbol{\rho}_a = \mathrm{Tr}_b\,\boldsymbol{\rho} = \sum_m \sum_{\alpha\beta} |\alpha\rangle\langle\beta| \rho_{\alpha\beta mm} = \sum_{\alpha\beta} |\alpha\rangle\langle\beta| \left(\sum_m \rho_{\alpha\beta mm} \right)$$

Here we define,

$$\sum_m \rho_{\alpha\beta mm} = \rho^b_{\alpha\beta}$$

Then we have,

$$\boldsymbol{\rho}_a = \sum_{\alpha\beta} \rho^b_{\alpha\beta} |\alpha\rangle\langle\beta|$$ **and** $$\boldsymbol{\rho}_a^2 = \sum_{\alpha\beta\gamma} \rho_{\alpha\gamma}\rho_{\gamma\beta} |\alpha\rangle\langle\beta|$$

Condition for coherence (pure state)

$$\boldsymbol{\rho}_a = \boldsymbol{\rho}_a^2 \quad \Longleftrightarrow \quad \rho^b_{\alpha\beta} = \sum_\gamma \rho_{\alpha\gamma}\rho_{\gamma\beta}$$

Example of decoherence

R: right polarized

L: left polarized

When $$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\alpha\rangle|R\rangle + e^{i\theta}|\beta\rangle|L\rangle\right)$$

then

$$\rho = |\psi\rangle\langle\psi|$$

$$= \frac{1}{2}\left(|\alpha\rangle|R\rangle\langle R|\langle\alpha| + |\beta\rangle|L\rangle\langle L|\langle\beta| + e^{-i\theta}|\alpha\rangle|R\rangle\langle L|\langle\beta| + e^{i\theta}|\beta\rangle|L\rangle\langle R|\langle\alpha|\right)$$

therefore

$$\mathop{\mathrm{Tr}}_{\alpha\beta}\rho = \frac{1}{2}\left(|R\rangle\langle R| + |L\rangle\langle L|\right)$$ ⇒ ***Non-polarized light***

$$\begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}^2 = \begin{pmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{pmatrix}$$

Pure state

$$\begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}^2 = \begin{pmatrix} 1/4 & 0 \\ 0 & 1/4 \end{pmatrix}$$

Mixed state

Conclusion of density matrix consideration:

Partial observation of the system can reduce the coherence in subspace.

Examples:

1) If we observe light coming from *one slit* in the Young's double slit experiment, then no interference.

2) If we do not observe the photon field in the photon-matter interaction, the expectation value of the dipole moment of the matter is zero. (Appendix 2)

Glauber's coherent satate

$$|\alpha\rangle = \exp\left(-\frac{|\alpha|^2}{2}\right)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle$$

$$a|\alpha\rangle = \alpha|\alpha\rangle$$

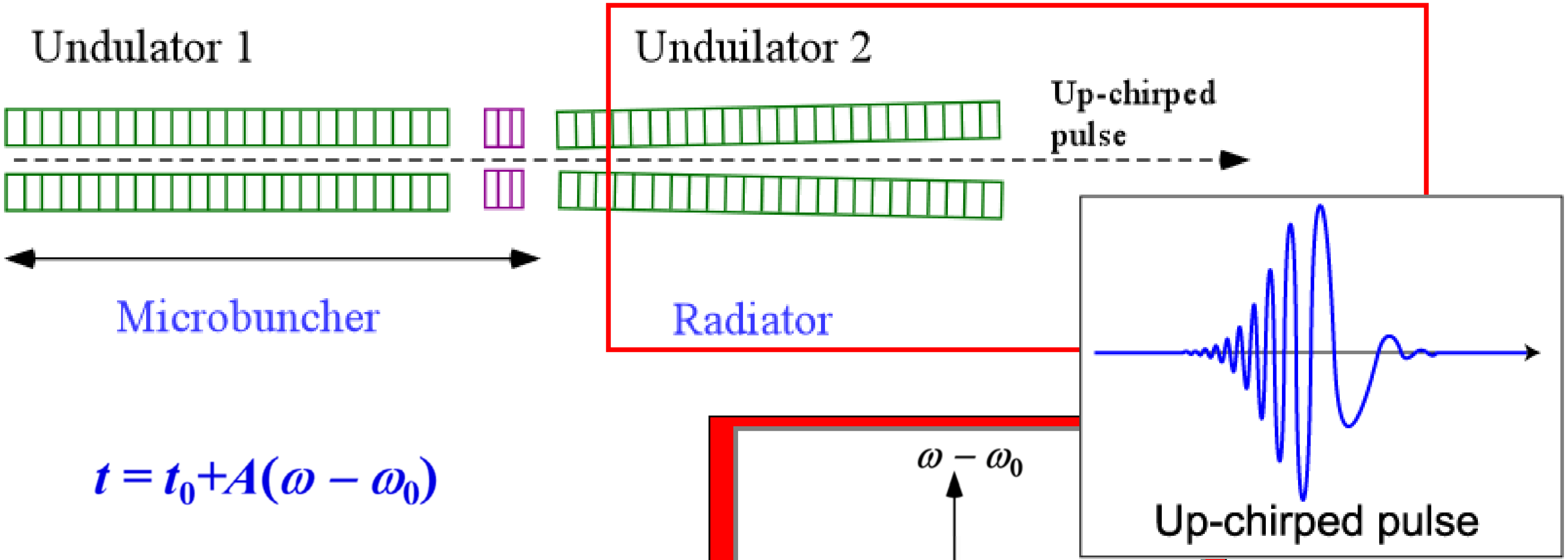
$$E \propto a^* + a$$

represents a classical electromagnetic wave, lasers.

Expectation value of the electric field: $\sin\omega t$

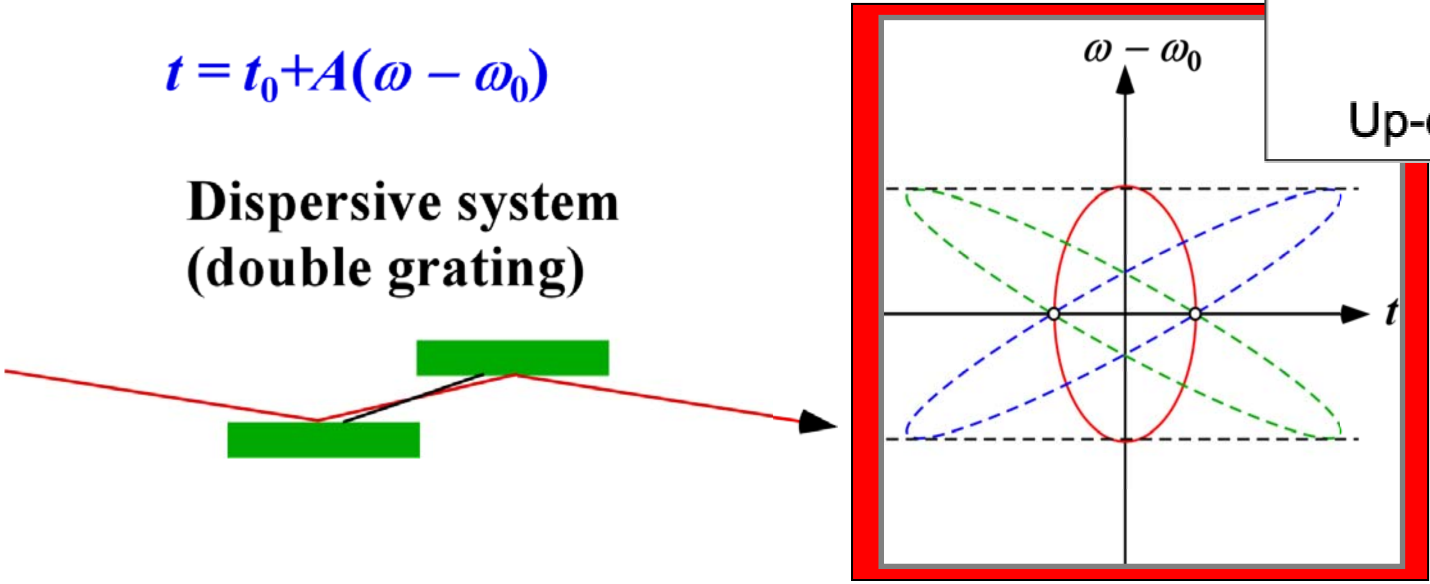
Outlook

Producrion of ultrashort pulse < 1 fsec

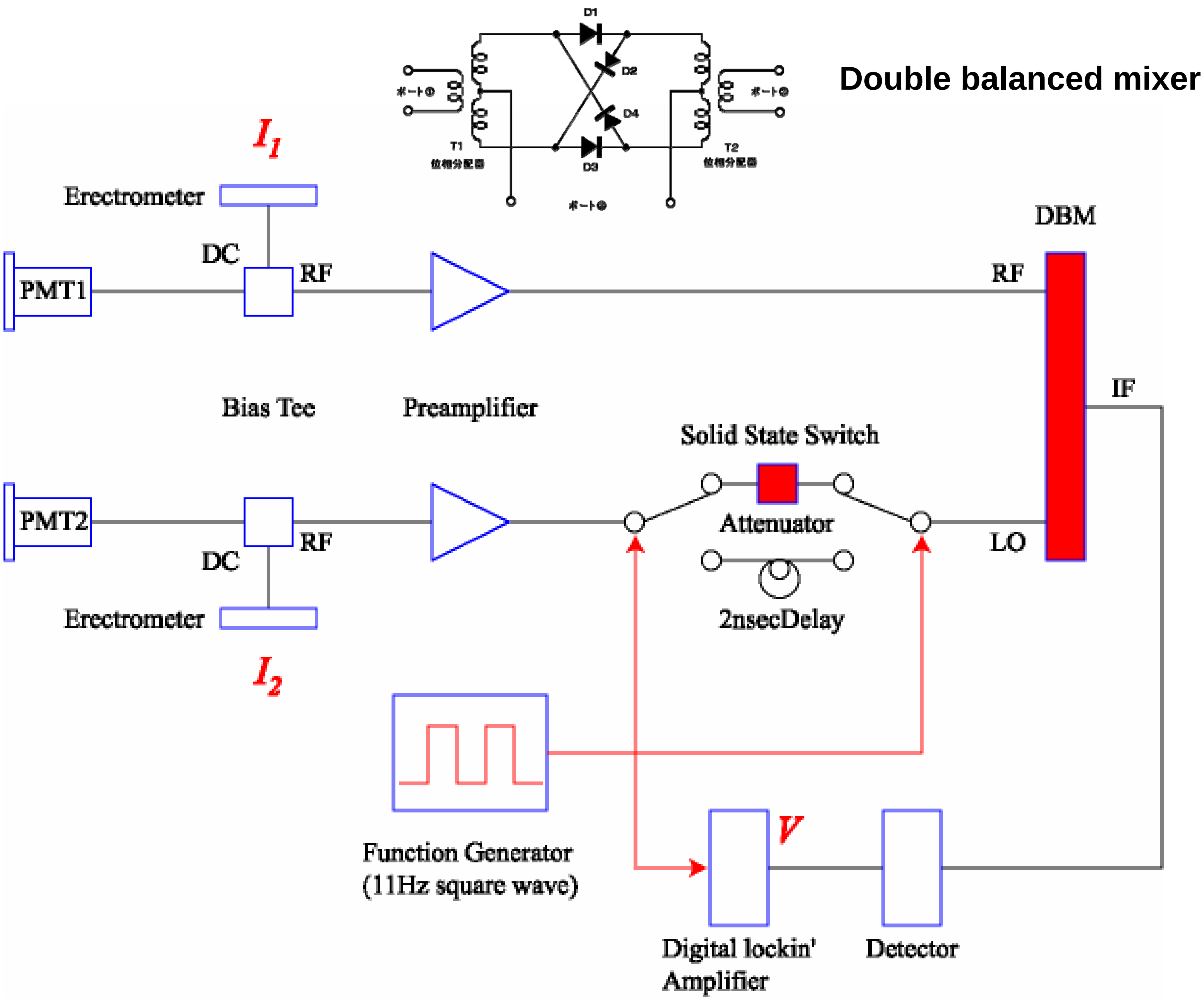


$$t = t_0 + A(\omega - \omega_0)$$

Dispersive system
(double grating)

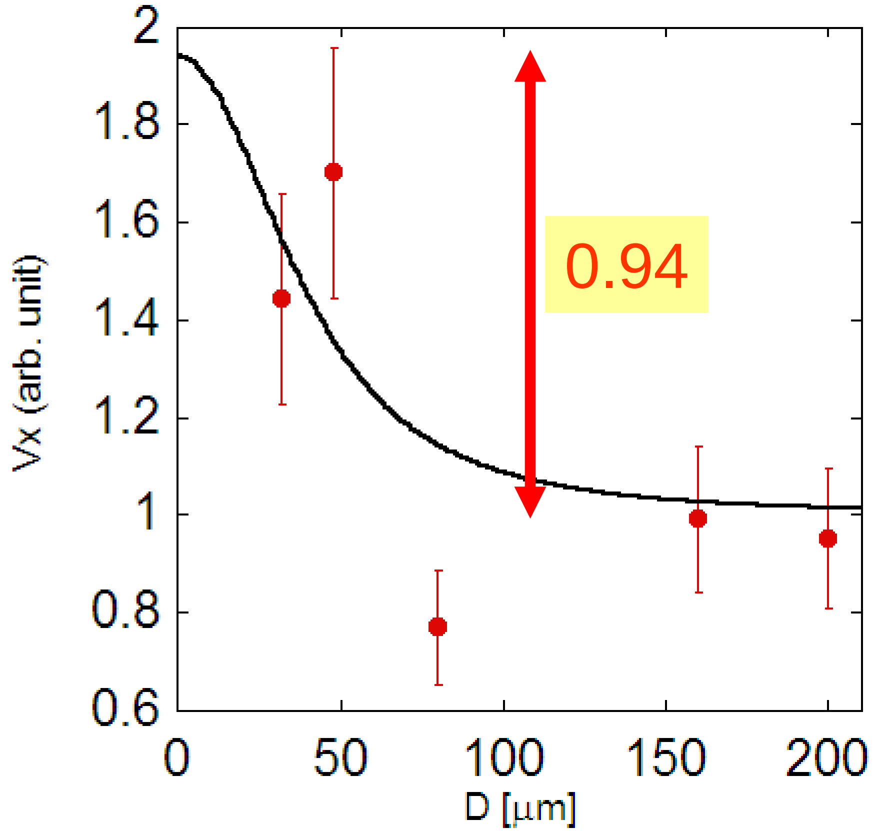


Electronics of the modulation technique to detect correlation

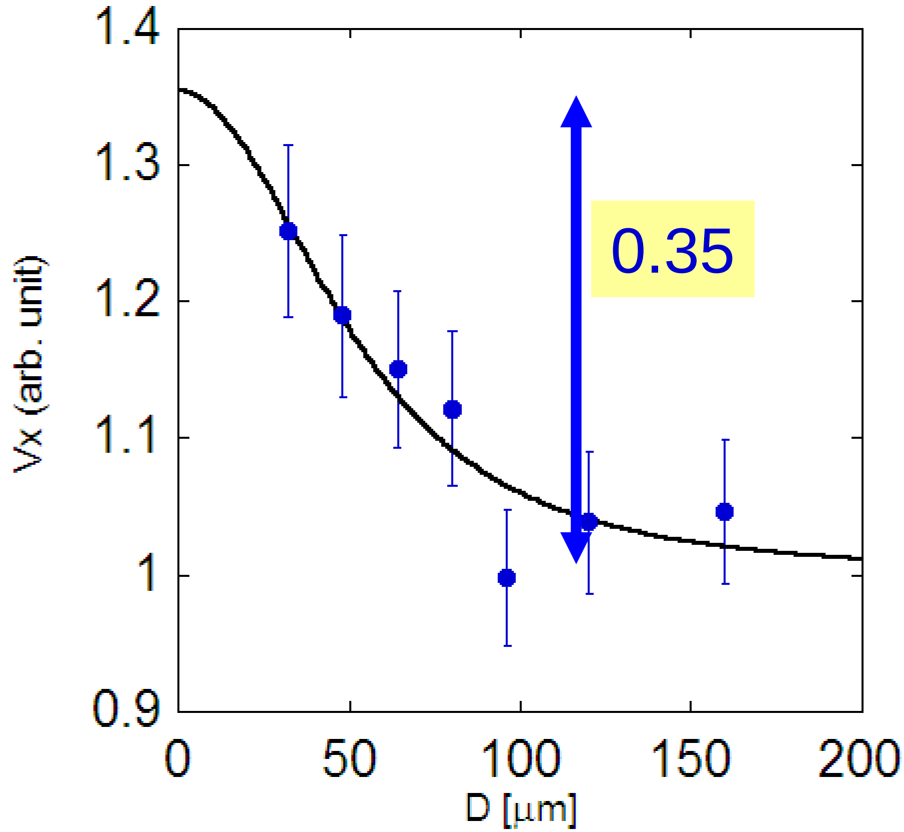


Results of two- photon correlation

Without taper (non-chirped pulse)



With taper (up-chirped pulse)



Correlation rate 1 : 0.38(±0.20) → Compressed to 38%

Summary

First-order coherence

1) First-order coherence depends on how we observe the light.

2) First-order coherence can be improved with sacrifice of intensity.

   The loss of intensity is smaller when the source has smaller emittance.

3) First-order spatial coherence is easily observed in Young's experiments.

4) The idea of first-order spatial coherence can be applied to the $\omega - t$ space.

5) Observation of a part of the system could reduce the coherence., corresponding to tracing out the density matrix in a sub-space.

Second-order coherence

1) Measurement of two-photon correlation gives information of photon statistic and the wave packet length of a photon.

2) Using a tapered undulator and a double grating system, the wave packet length can be compressed.

Appendix 1: Time evolution

Hamiltonian: $\mathrm{H} = \mathrm{H}_a + \mathrm{H}_b + \mathrm{H}_{ab}$

Eigen energies of H_a in a : E_α

Eigen energies of H_b in b : E_k

$$\left|\alpha\right\rangle = \exp\left(-\frac{i}{\hbar} E_\alpha t\right) \qquad \left|k\right\rangle = \exp\left(-\frac{i}{\hbar} E_k t\right)$$

$$\frac{d}{dt}\left\langle \mathrm{A} \right\rangle = \mathrm{Tr}\left(\dot{\boldsymbol{\rho}}\, \mathrm{A}\right) = \sum_l \sum_{\beta\gamma} \left\{ \dot{\rho}_{\gamma\beta ll} + \frac{i}{\hbar}\left(E_\gamma - E_\beta\right)\rho_{\gamma\beta ll} \right\} \left\langle \beta \middle| \mathrm{A} \middle| \gamma \right\rangle$$

If A does not observe subspace b,

$$\frac{d}{dt}\left\langle \mathrm{A} \right\rangle = \sum_{\beta\gamma} \left\{ \dot{\rho}^b_{\gamma\beta} + \frac{i}{\hbar}\left(E_\gamma - E_\beta\right)\rho^b_{\gamma\beta} \right\} \left\langle \beta \middle| \mathrm{A} \middle| \gamma \right\rangle$$

Appendix 2: broken symmetry operator

space a: electronic system,
(creation, annihilation operators) c_α^+ c_α

space b: bosonic system a_k^+ a_k

Interaction Hamiltonian:

$$\mathrm{H}_{ab} = c_\alpha^+ c_\beta a_k \langle \alpha | \mathrm{A} | \beta \rangle + \mathrm{c.c.}$$

Assuming correlation (*entanglement*)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|e\rangle|n\rangle + |g\rangle|n+1\rangle\right)$$

Then matrix element of A is,

$$\rho_{\alpha\beta}^{b} = \sum_{l} \rho_{\alpha\beta ll} = 0 \qquad (\alpha \neq \beta)$$

$\langle \alpha | A | \alpha \rangle = 0$ and $\langle \mathrm{A} \rangle = 0$ *"dipole moment" is zero.*