Light Source I

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CONTENTS

Introduction

SR Facility and Light Source

- SR: Definition
  - Electromagnetic wave emitted by a charged particle deflected by a magnetic force
- SR Facility
  - Accelerators to generate a high-energy electron beam
  - **Magnetic devices (SR light source) to generate intense SR**
  - Optical elements (monochromators, mirrors,..)
  - Experimental stations

SR as a Probe for Research

- SR has a lot of advantages over other conventional light sources
  - Highly collimated (laser-like)
  - Wavelength tunability
  - Polarization
  - .....
- However, the total radiation power does not differ significantly.

Comprehensive understanding of SR (and light source) is required for efficient experiments.

Topics in This Lecture (1)

- Fundamentals of Light and SR
  - General description
  - Physical quantity of light
  - Uncertainty of light: Fourier and diffraction limits
  - SR: Light from a moving electron
- Overview of SR Light Source
  - Types of light sources
  - Magnet configuration
- Characteristics of SR (1)
  - Radiation from bending magnets

Topics in This Lecture (2)

- Characteristics of SR (2)
  - Radiation from wigglers
  - Radiation from undulators
    - Electron motion in the undulator
    - Fundamental energy
    - Spectral function
    - Spatial profile of radiation
    - Higher harmonics
- Practical and Useful Knowledge on SR
  - Finite emittance and energy spread
  - Heat load and photon flux
  - Definition of undulators and wigglers

Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

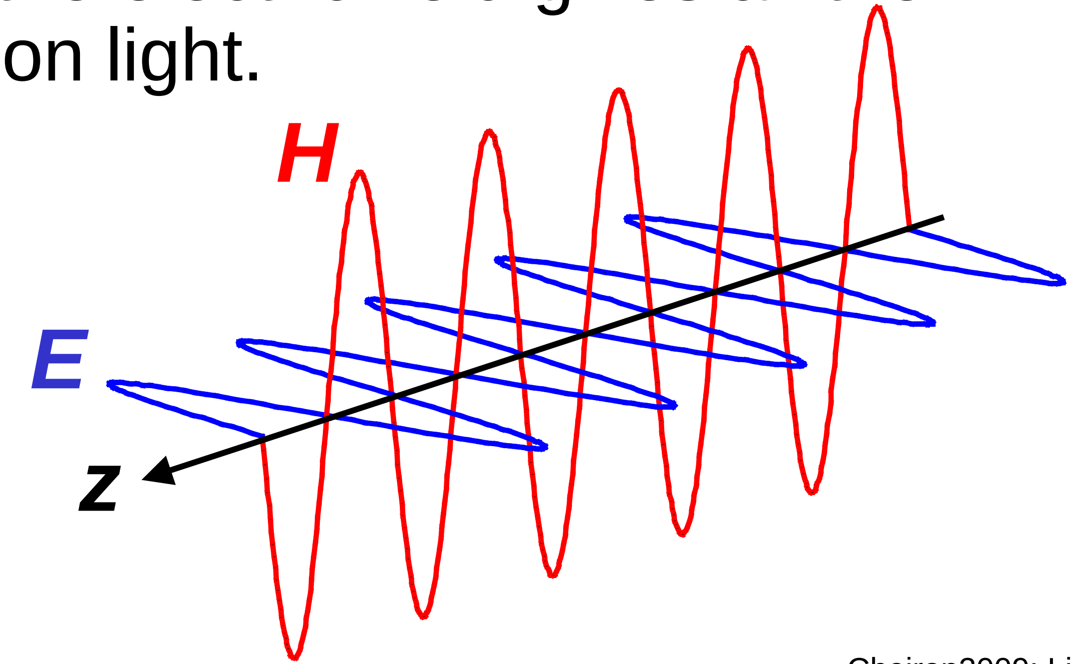
Light as an Electromagnetic Wave

- Electric and magnetic fields can propagate in vacuum as a transverse wave, with a relation

$$\boldsymbol{H} = \frac{1}{\boxed{\mu_0 c}} \hat{\boldsymbol{z}} \times \boldsymbol{E}$$

vacuum impedance (377Ω)

- Analysis of the electric field gives all the information on light.



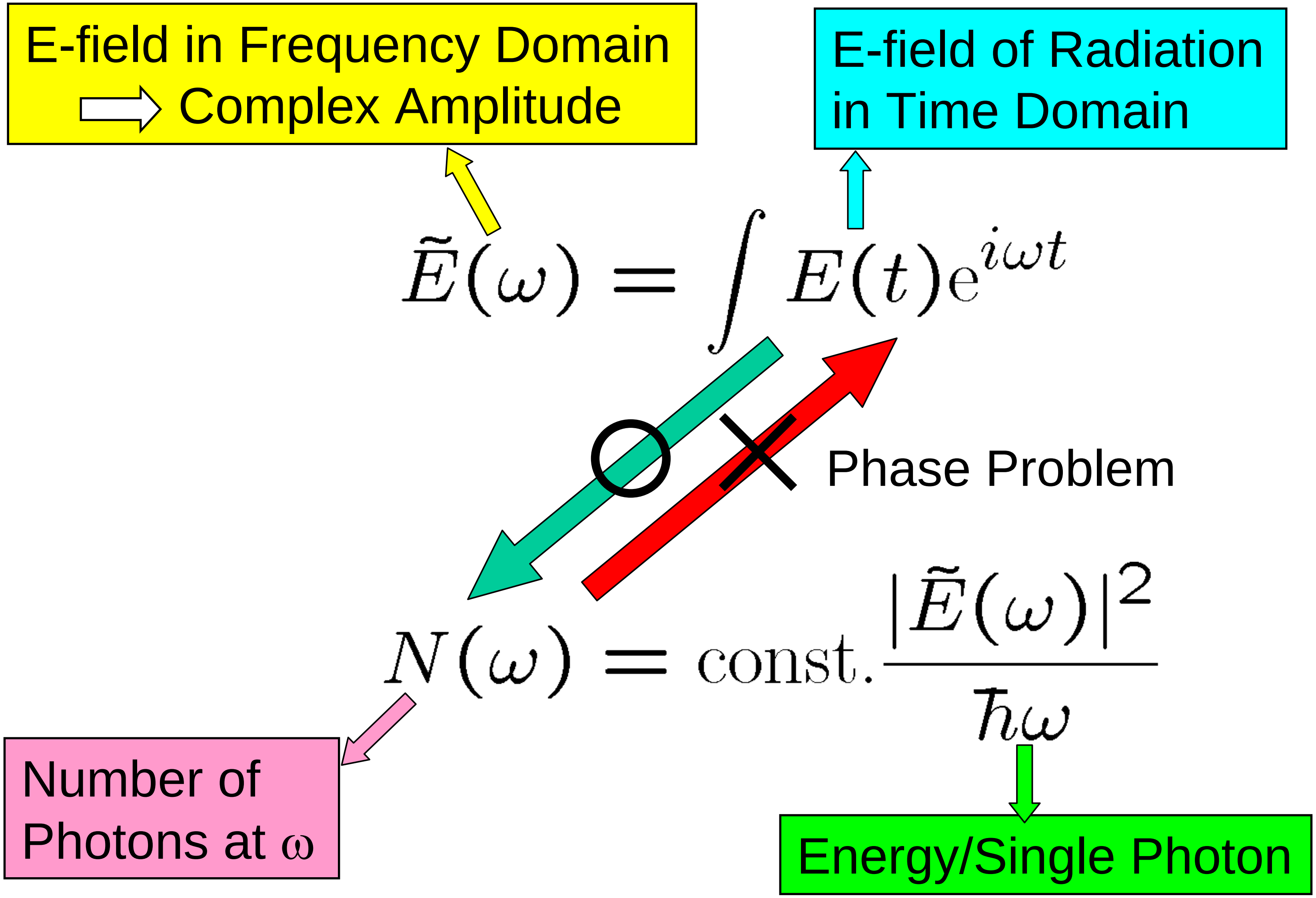
Light as a Photon

- In the quantum theory, light is not only an electromagnetic wave but also a particle, or a photon.
- The energy of a photon with a wavelength of λ is given by

$$E_{photon} = \hbar\omega = \frac{hc}{\lambda}$$

- For example, a photon with the wavelength of 0.1nm (1Å=10^{-10}m) is calculated as 12.4 keV.

Relation Between EM Wave and Photon



E-field in Frequency Domain
⇨ Complex Amplitude

E-field of Radiation
in Time Domain

$$\tilde{E}(\omega) = \int E(t)\mathrm{e}^{i\omega t}$$

Phase Problem

$$N(\omega) = \mathrm{const.}\frac{|\tilde{E}(\omega)|^2}{\hbar\omega}$$

Number of
Photons at ω

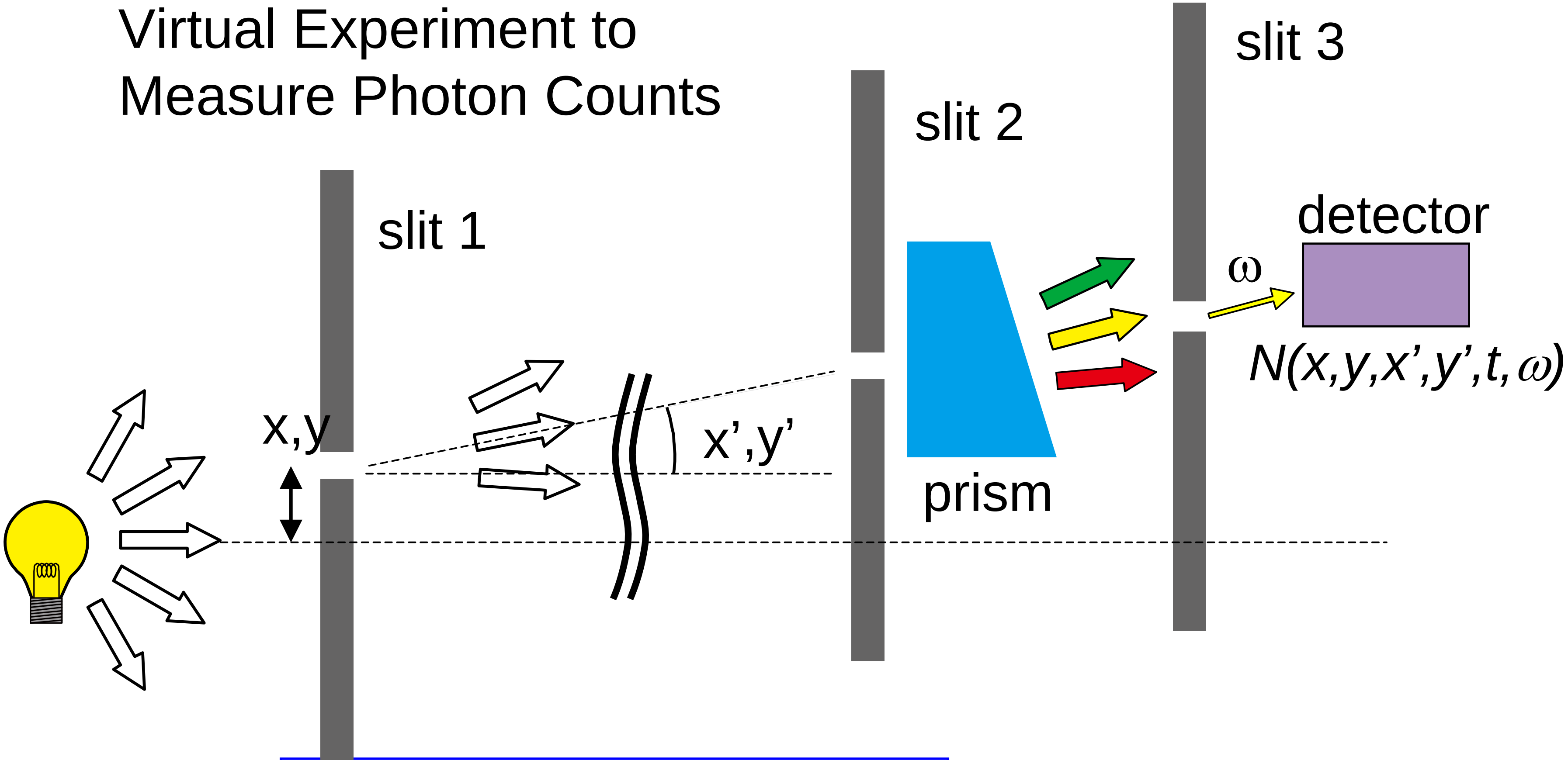
Energy/Single Photon

Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

Phase Space

Virtual Experiment to Measure Photon Counts



$$n = \boxed{\frac{N(x,y,x',y',\omega,t)}{\Delta x\,\Delta y\,\Delta x'\,\Delta y'\,\Delta t\,\Delta\omega/\omega}} = \frac{N}{\boxed{\Delta\Omega}}$$

Av. Photon Density

Volume in 6-D Phase Space

Brilliance (Brightness)

- Brilliance (photons/sec/mm^2/mrad^2/0.1%B.W.) is defined as the photon density in the 6D phase space, i.e.,

$$B = \lim_{\Delta\Omega\to 0} n = \frac{d^6 N(x, y, x', y', t, \omega)}{dx\,dy\,dx'\,dy'\,dt\,d\omega/\omega}$$

- In practice, $\Delta\Omega$ can never be 0 due to uncertainty of light, thus brilliance is not a physical quantity that can be actually measured.

Photon Flux and Flux Density

- Removing the 1st slit gives the angular flux density (photons/sec/mrad^2/0.1%B.W), i.e.,

$$\frac{d^2F}{dx'dy'} = \iint B dx dy$$

- Removing the 1st & 2nd slits gives the total flux (photons/sec/0.1%B.W), i.e.,

$$F = \iiiint B dx dy dx' dy'$$

- Estimation of number of photons to be delivered to the sample.

Radiation Power and Power Density

- Removing the 1st & 3rd slits gives the angular power density (W/mrad^2), i.e.,

$$\frac{d^2P}{dx'dy'} = 10^3 Q_e \hbar \int \frac{d^2F}{dx'dy'} d\omega$$

conversion from photons/sec/0.1%B.W. to W

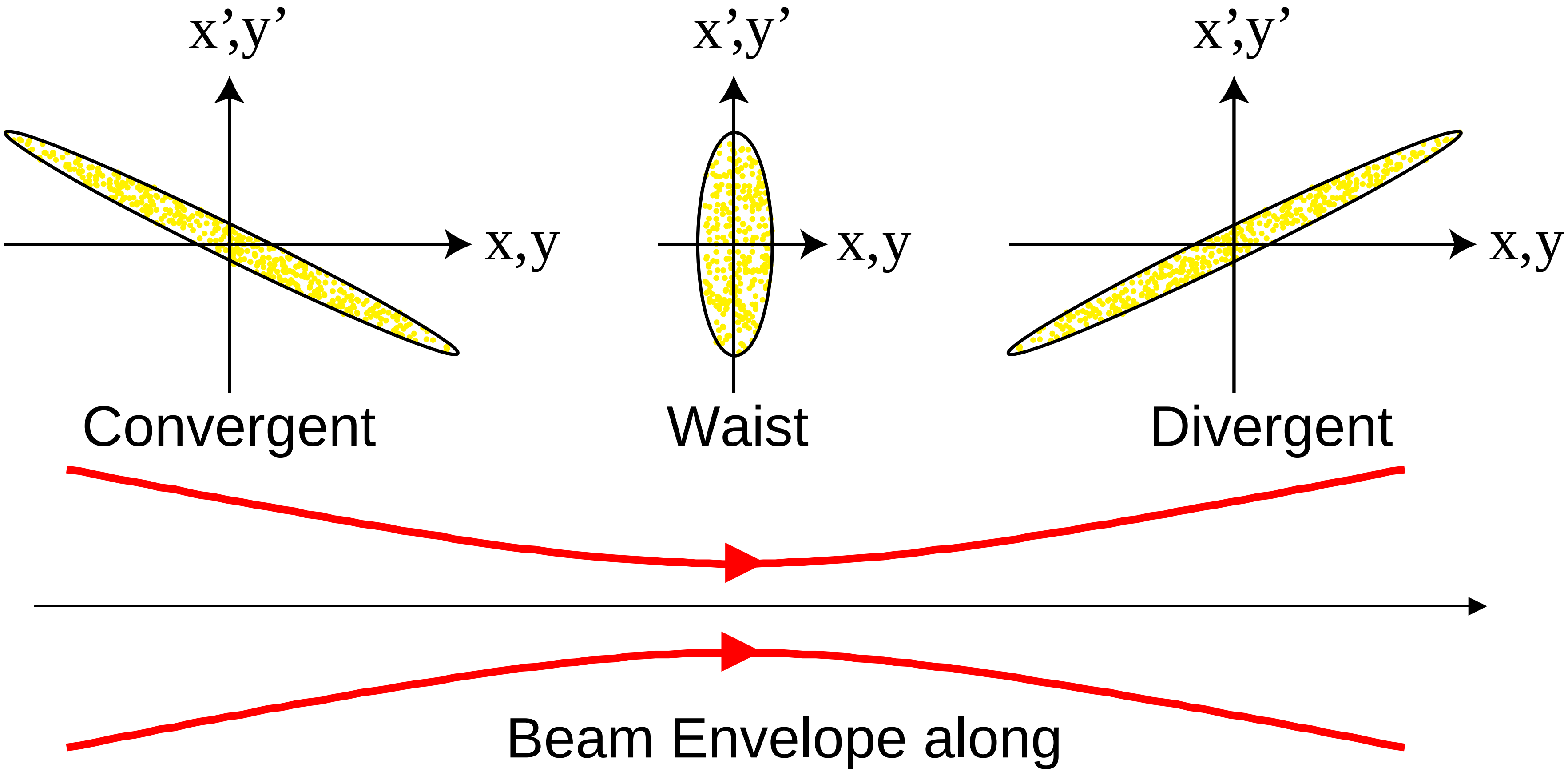
- Removing all the slits gives the total power (W), i.e.,

$$P = 10^3 Q_e \hbar \iiint \frac{d^2F}{dx'dy'} d\omega dx' dy'$$

- Estimation of heat load on BL components.

Photons in 4D Phase Space

- Photon distribution in the 4-D phase space at different longitudinal positions.



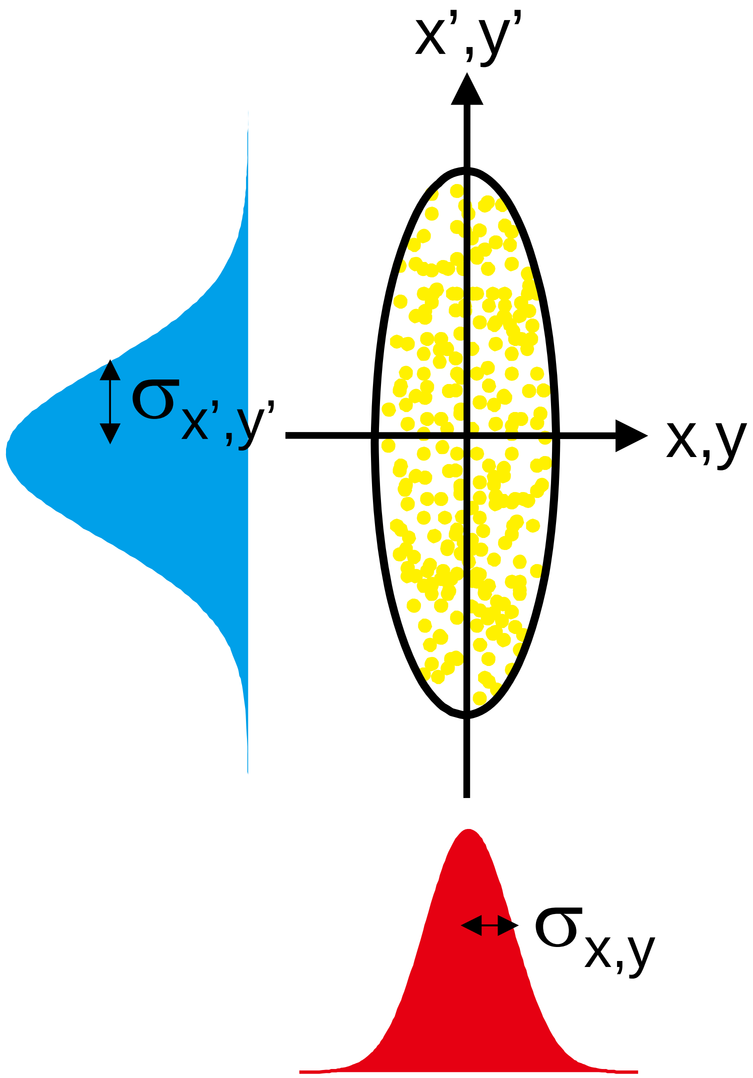
Convergent

Waist

Divergent

Beam Envelope along Propagation Axis

Beam Size, Divergence, Emittance



- Beam size ($\sigma_{x,y}$) is defined as the beam envelope at the beam waist position.
- Angular divergence ($\sigma_{x',y'}$) is constant along the axis of propagation, as far as no optical elements are present.
- Emittance ($\varepsilon_x, \varepsilon_y$) is defined as $\sigma_{x,y} \times \sigma_{x',y'}$, which is equal to the area of the phase ellipse divided by π.

Estimation of On-Axis Brilliance

- Gaussian Approximation of *B*

$$B(x,y,x',y') = B_0 \exp\left(-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} - \frac{x'^2}{2\sigma_{x'}^2} - \frac{y'^2}{2\sigma_{y'}^2}\right)$$

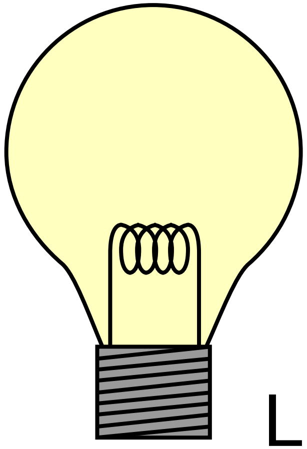
$$\left.\frac{d^2F}{dx'dy'}\right|_0 = \iint B\,dxdy = 2\pi\sigma_x\sigma_y B_0$$

$$F = \iiiint B\,dxdydx'dy' = 4\pi^2\sigma_x\sigma_y\sigma_{x'}\sigma_{y'}B_0$$

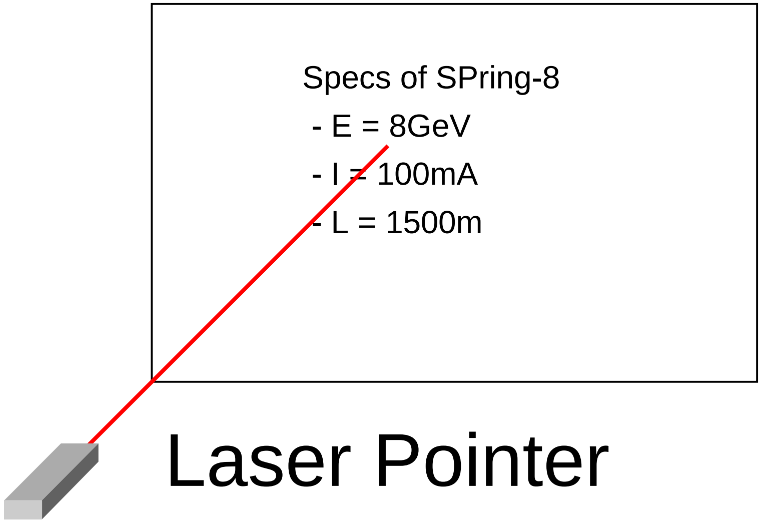
$$\boxed{B_0 = \left.\frac{d^2F}{dx'dy'}\right|_0 \frac{1}{2\pi\sigma_x\sigma_y} = \frac{F}{4\pi^2\sigma_x\sigma_y\sigma_{x'}\sigma_{y'}}}$$

estimation of on-axis brilliance
with measurable values

Example: Laser Pointer and Light Bulb



Light Bulb

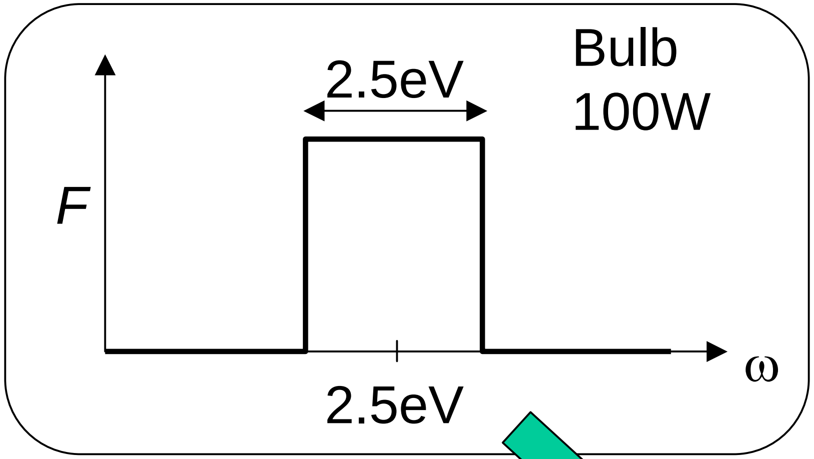


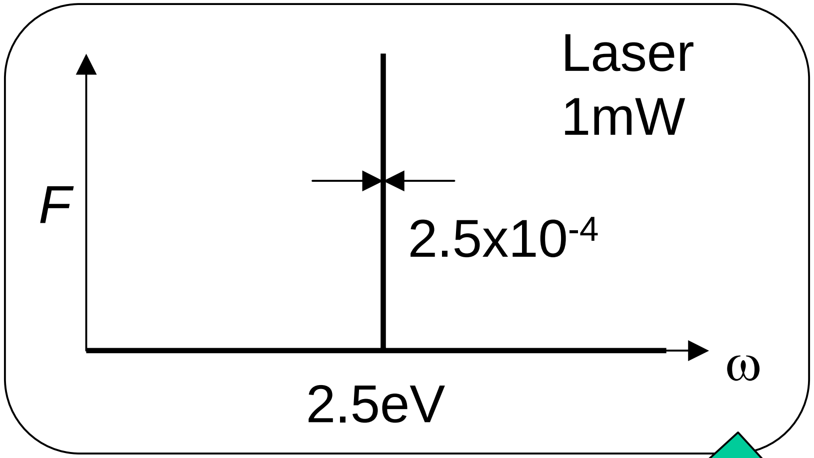
Specs of SPring-8
- E = 8GeV
- I = 100mA
- L = 1500m

Laser Pointer

	Bulb	Laser
$\lambda_r(\hbar\omega_r)$	500nm (2.5eV)	
Band Width:	100 (%)	0.01 (%)
Total Power: W	100 W	1 mW
Angular Divergence	4π rad	1 mrad^2
Beam Size:	1 cm^2	1 mm^2

Example: Photon Flux





$$P = 10^3 Q_e \hbar \int F d\omega$$

	Bulb	Laser
Total Power	100 W	1 mW
Band Width	100 (%)	0.01 (%)
	2.5 eV	2.5×10^{-4} eV
Flux@500nm	10^{17}	10^{16}

Example: Brilliance

$$B = F/(4\pi^2 \sigma_x \sigma_y \sigma_{x'} \sigma_{y'})$$

	Bulb	Laser
Total Power	100 W	1 mW
Flux@500nm	10^{17}	10^{16}
Angular Divergence	4π rad	1 mrad^2
Beam Size	1 cm^2	1 mm^2
Brilliance	10^7	10^{14}

- **Laser is an ideal light source in terms of brilliance, but ...**
- **No lasers exist in x-ray region**
    - ➢**Synchrotron Radiation**

Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

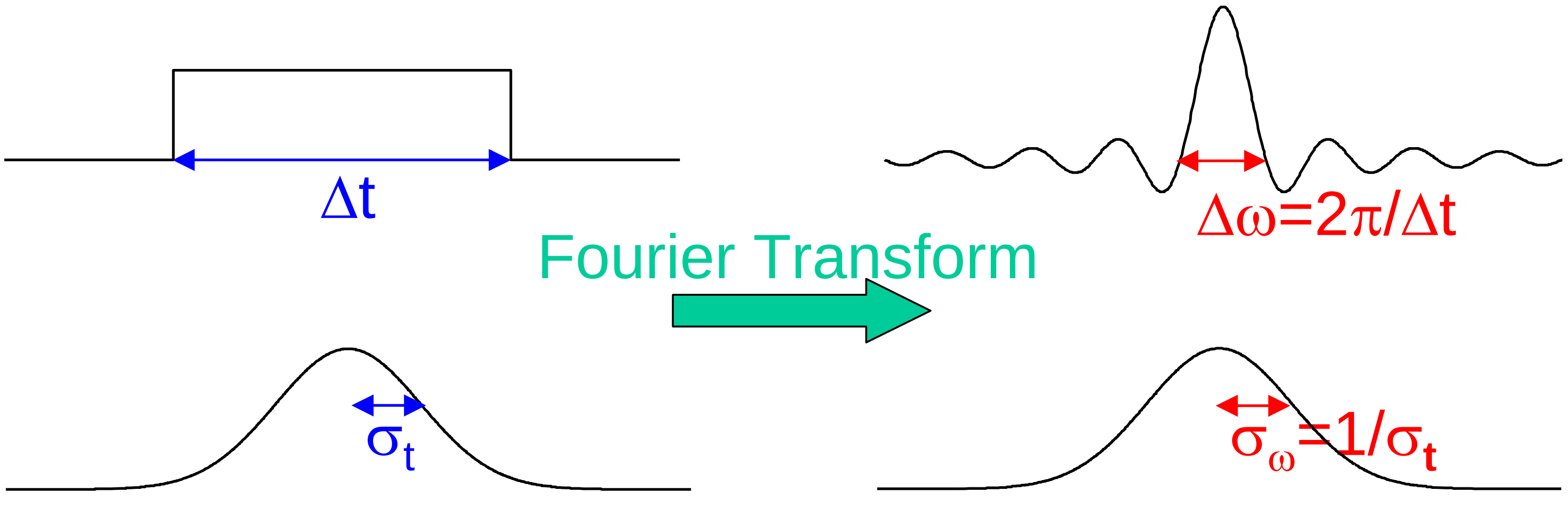
Uncertainty of Light

- The photon distribution in the 6D phase space (x,y,x',y',t,ω) gives us the full information on the properties of SR.
- Due to wave nature of light, however, we have two uncertainty relations to take care, which are well characterized by the Fourier transform.
- These relations imposes two restrictions on SR, Fourier and Diffraction limits.

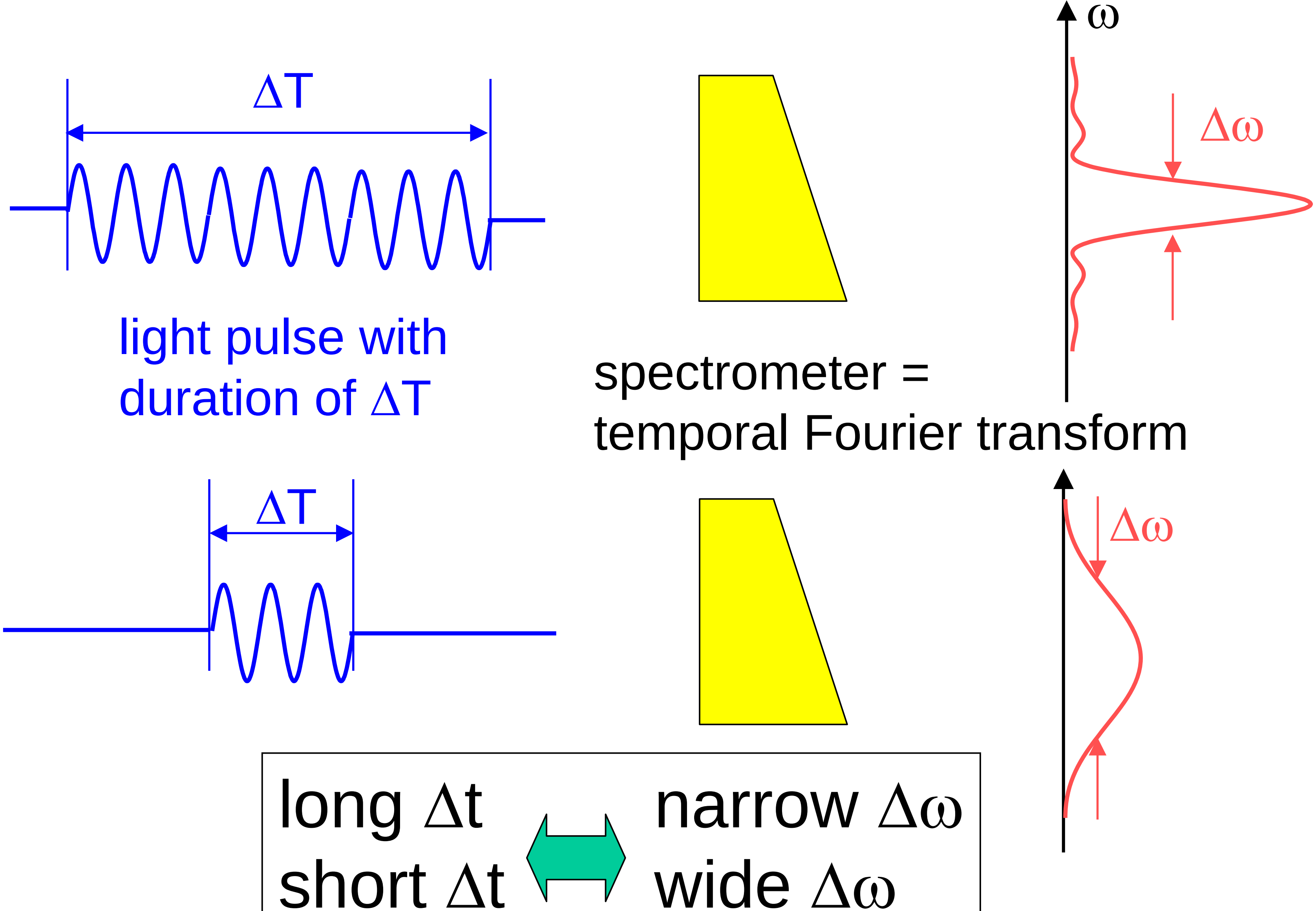
Fourier Transform: Example

Important Fourier Transform in SR Formulae

$F(t)$	$f(\omega) = \int_{-\infty}^{\infty} F(t) \mathrm{e}^{i\omega t}$
$\begin{cases} 1/\Delta t; & -\Delta t/2 \leq t \leq \Delta t/2 \\ 0; & t < -\Delta t/2, \Delta t/2 < t \end{cases}$	$\dfrac{\sin \omega \Delta t/2}{\omega \Delta t/2} \equiv \mathrm{sinc}(\omega \Delta t/2)$
$\dfrac{1}{\sqrt{2\pi}\sigma_t} \exp\left(-\dfrac{t^2}{2\sigma_t^2}\right)$	$\exp\left(-\dfrac{\omega^2 \sigma_t^2}{2}\right)$



Temporal Fourier Transform



light pulse with
duration of ΔT

spectrometer =
temporal Fourier transform

long Δt ⟷ narrow Δω
short Δt ⟷ wide Δω

Fourier Limit of Light

- Temporal Fourier transform imposes

$$\Delta\omega\Delta t \geq 1$$

- Uncertainty of light in the (ω,t) plane.
- When equality holds, light is said to be Fourier-limited, or temporally coherent.
- Important to understand the spectral properties of SR.

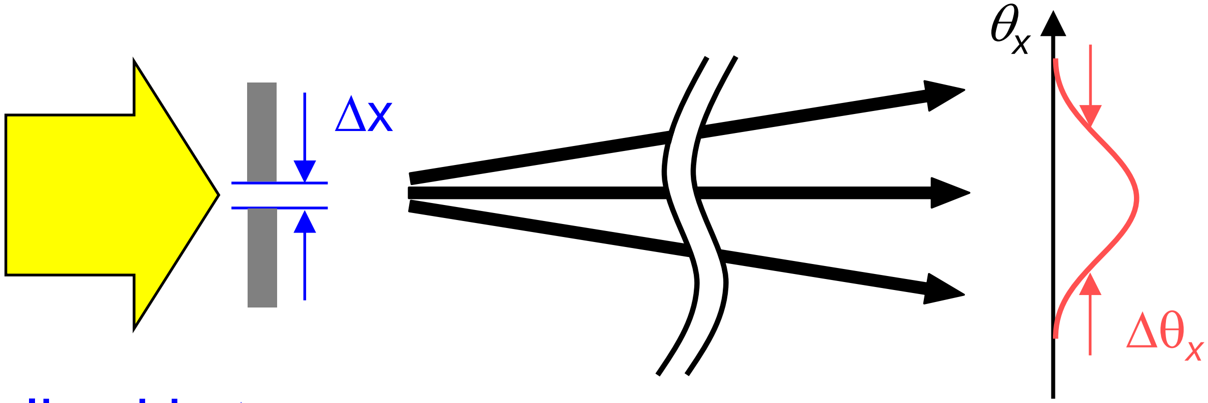
Monochromatic Light

- Ideally, “monochromatic” means

$$\Delta\omega = 0 \;\&\; \Delta t = \infty$$

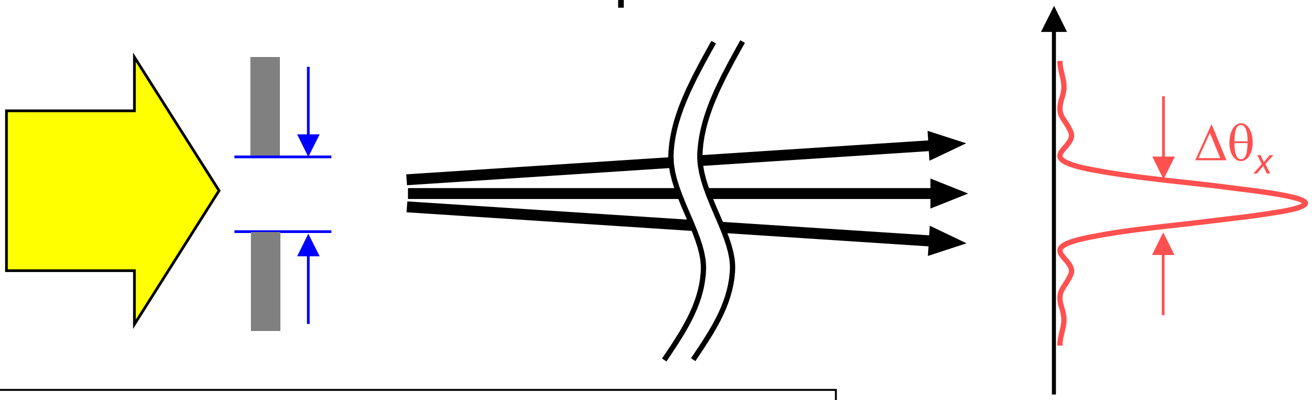
- In practice, perfectly monochromatic light does not exist, but all kinds of light are partially monochromatic.
- Temporal Fourier transform is to decompose the practical light into components of perfectly monochromatic light.

Spatial Fourier Transform



slit with Δx
= beam size

diffraction pattern in the far region
= spatial Fourier transform



wide Δx ⟷ narrow $\Delta\theta_x$

narrow Δx ⟷ wide $\Delta\theta_x$

Diffraction Limit of Light (1)

Photon Beam with 1D Gaussian Profile

$$|\tilde{E}(x)|^2 \propto \exp(-x^2/2\sigma_x^2)$$ Photon Spatial Profile

$$\tilde{E}(x) \propto \exp(-x^2/4\sigma_x^2)$$ E-field Spatial Profile

Fourier transform

$$\tilde{E}(k_x) \propto \exp(-k_x^2\sigma_x^2)$$ E-field k-vector Profile

$$|\tilde{E}(k_x)|^2 \propto \exp(-2\sigma_x^2 k_x^2)$$ Photon k-vector Profile

$$\equiv \exp(-\theta_x^2/2\sigma_{x'}^2)$$ Photon Angular Profile

$$k_x = (2\pi/\lambda)\theta_x$$

$$\sigma_{x'}\sigma_x = \frac{\lambda}{4\pi}$$ Natural emittance of light with wavelength λ

Diffraction Limit of Light (2)

- Spatial Fourier transform imposes

$$\sigma_{x,y}\sigma_{x',y'} \geq \lambda$$

- Uncertainty of light in the (x,x’,y,y’) plane.
- When equality holds, light is said to be diffraction-limited, or spatially coherent.
- Important to derive beam size and angular divergence SR.

Plane Wave

- Ideally, “plane wave” means

$$\sigma_{x',y'} = 0 \;\&\; \sigma_{x,y} = \infty$$

- In practice, perfectly plane wave does not exist, but all kinds of light are partially plane-wave-like.
- Spatial Fourier transform is to decompose the practical light into components of perfectly plane wave.

Fundamentals of Light and SR

- General Description
- Physical Quantity of Light
- Uncertainty of Light
- SR: Light from a Moving Electron

SR: Light from a Moving Electron

- Unlike the ordinary light source (sun, light bulb,...), the light emitter of SR (electron) is ultra-relativistic.
- The characteristics of SR is thus quite different due to relativistic effects.
- What we have to take care is:
    1. Speed-of-light limit
    2. Squeezing of light pulse
    3. Conversion of the emission angles

Speed-of-Light Limit

Within the framework of relativity, the velocity of an electron never exceeds the speed of light.

$$v/c = \beta \;=\; \sqrt{1-\gamma^{-2}}$$

$$\sim\; 1-\frac{1}{2\gamma^2}$$

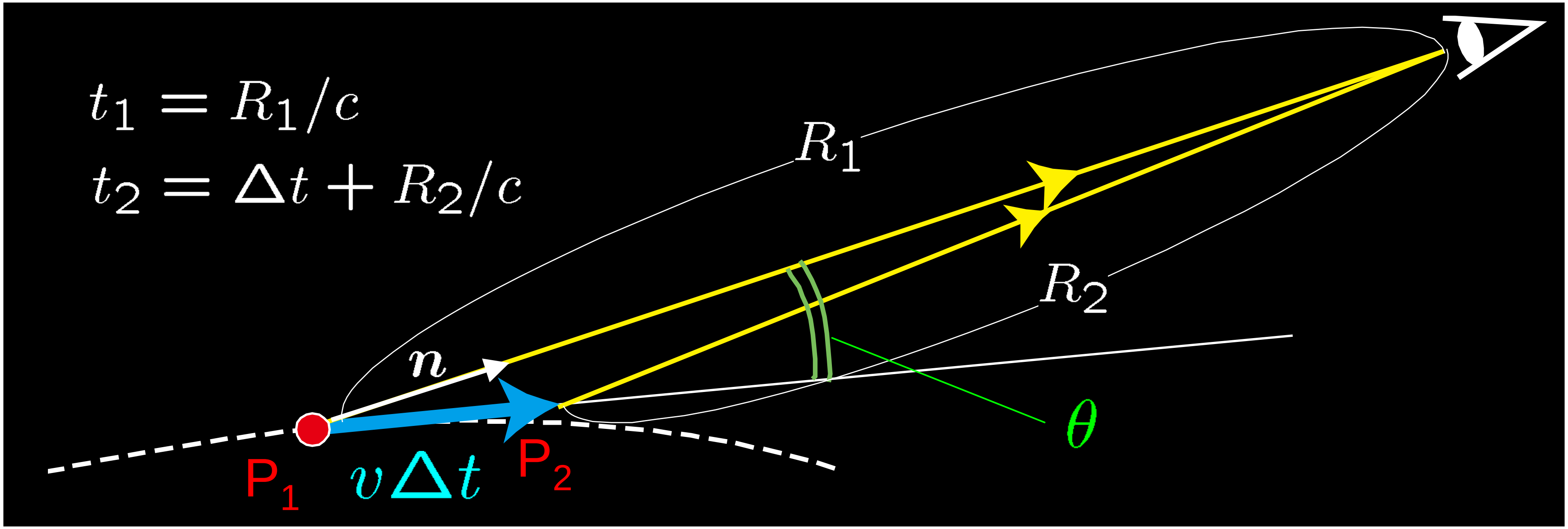
Energy	β
1MeV	0.941
10MeV	0.9988
100MeV	0.999987
8GeV	0.999999998

$$\gamma = \frac{E}{mc^2}$$

:Lorentz Factor

(relative electron energy, mc^2=0.511MeV)

Squeezing of Light Pulse Duration

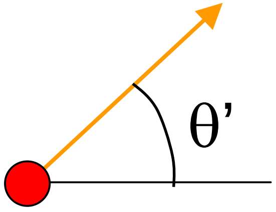


$$\begin{aligned} R_2 &= \sqrt{(R_1)^2+(v\Delta t)^2-2R_1 v\Delta t\cos\theta} \\ &\sim R_1 - (\boldsymbol{v}\cdot\boldsymbol{n})\Delta t \\ \Delta\tau &= t_2-t_1 = \Delta t + R_2/c - R_1/c \\ &= \Delta t\,\boxed{(1-\boldsymbol{\beta}\cdot\boldsymbol{n})} = \boxed{\frac{\Delta t}{2\gamma^2}} \end{aligned}$$

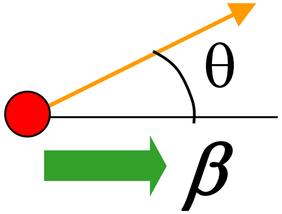
$\gamma>>1,\theta=0$

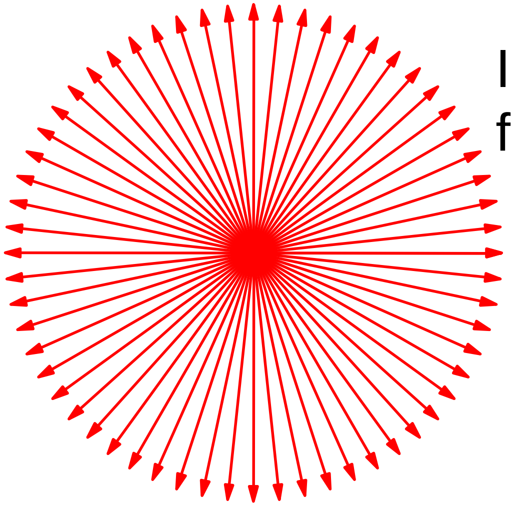
time squeezing

Conversion of Emission Angles

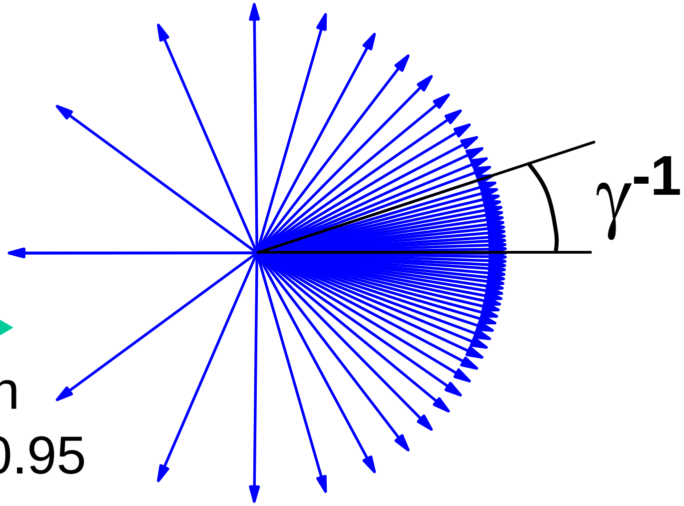


$$\theta = \tan^{-1}\left(\frac{\gamma^{-1}\sin\theta'}{\beta+\cos\theta'}\right)$$





Isotropic emission
from a rest electron



Emission from an
electron with $\beta=0.95$

Light emitted from a moving object ($\beta\sim1$) concentrates within γ^{-1}

SR from a High-Energy Electron

High energy electron

$$\beta \sim 1 = 1 - \frac{1}{2\gamma^2}$$

Large Time Squeezing

Short Wavelength

$$\theta \leq \gamma^{-1} \ll 1$$

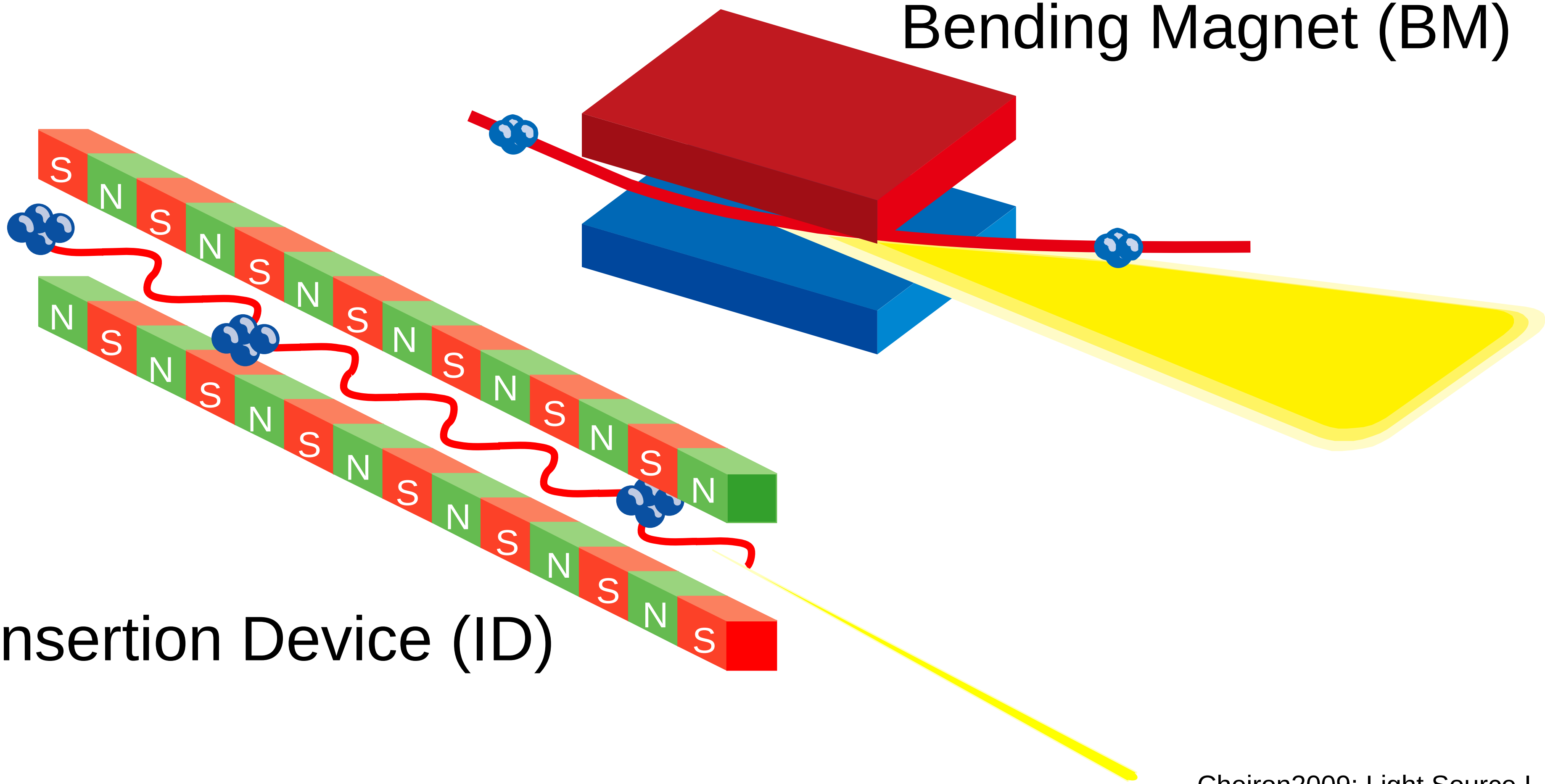
Radiation in the Forward Direction

High Directivity

Overview of SR Light Source

What is SR Light Source?

Magnets to deflect the electron beam and generate SR.



Bending Magnet (BM)

Insertion Device (ID)

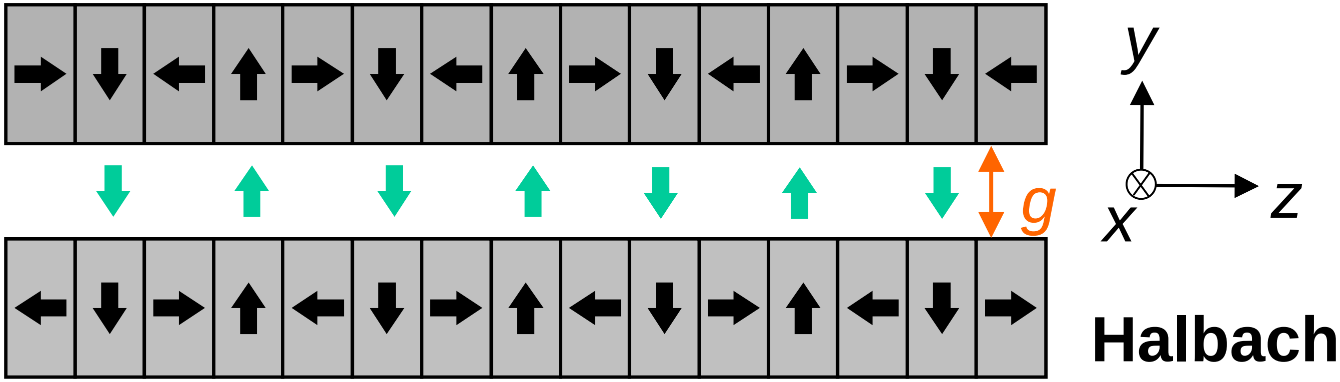
Bending Magnet

- One of the accelerator components in the storage ring.
- Generate uniform field to guide the electron beam into a circular orbit.
- EMs combined with highly-stable power supplies are adopted in most BMs due to stringent requirement on field quality and stability.
- Superconducting magnets are used in a few facilities in pursuit of harder x rays.

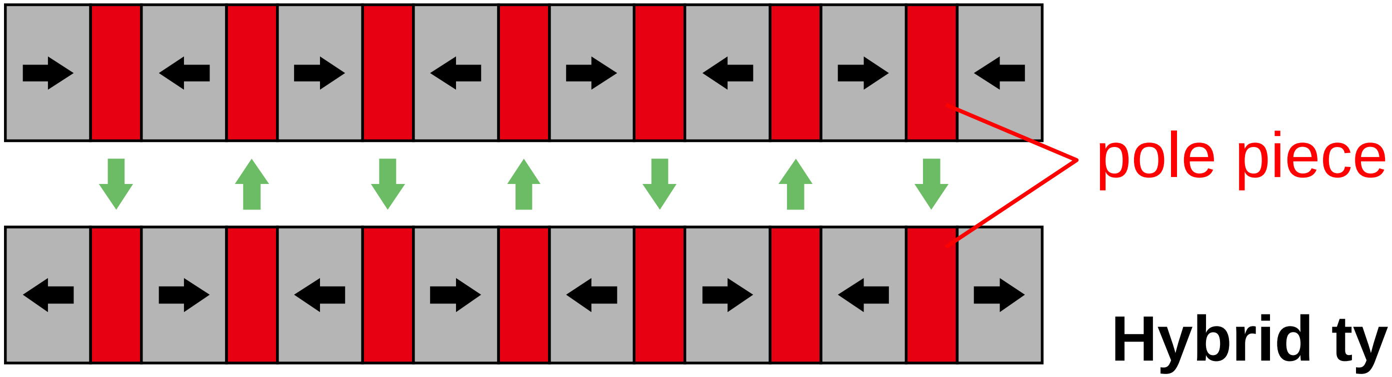
Insertion Device

- Installed (inserted) into the straight section of the storage ring between two adjacent BMs.
- Generate a periodic magnetic field to let the injected electron beam move along a periodic trajectory.
- Most IDs are composed of PMs, while EMs are used for special use such as helicity switching.
- Classified into wigglers and undulators.

Magnetic Circuit of IDs



Halbach type



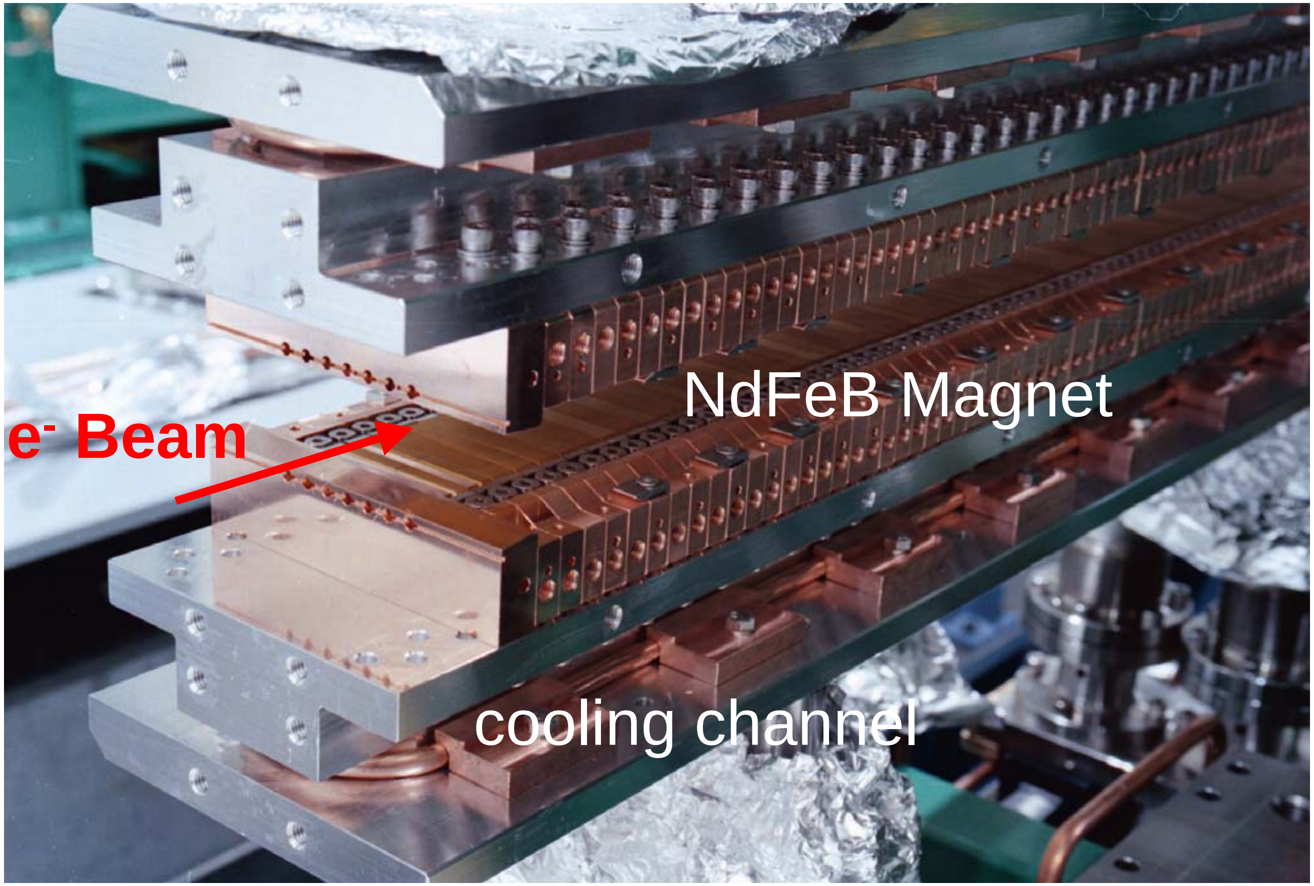
Hybrid type

In each type, a sinusoidal magnetic field is obtained:

$$B_y(z) \sim B_0(B_r, g/\lambda_u) \sin\left(\frac{2\pi z}{\lambda_u}\right)$$

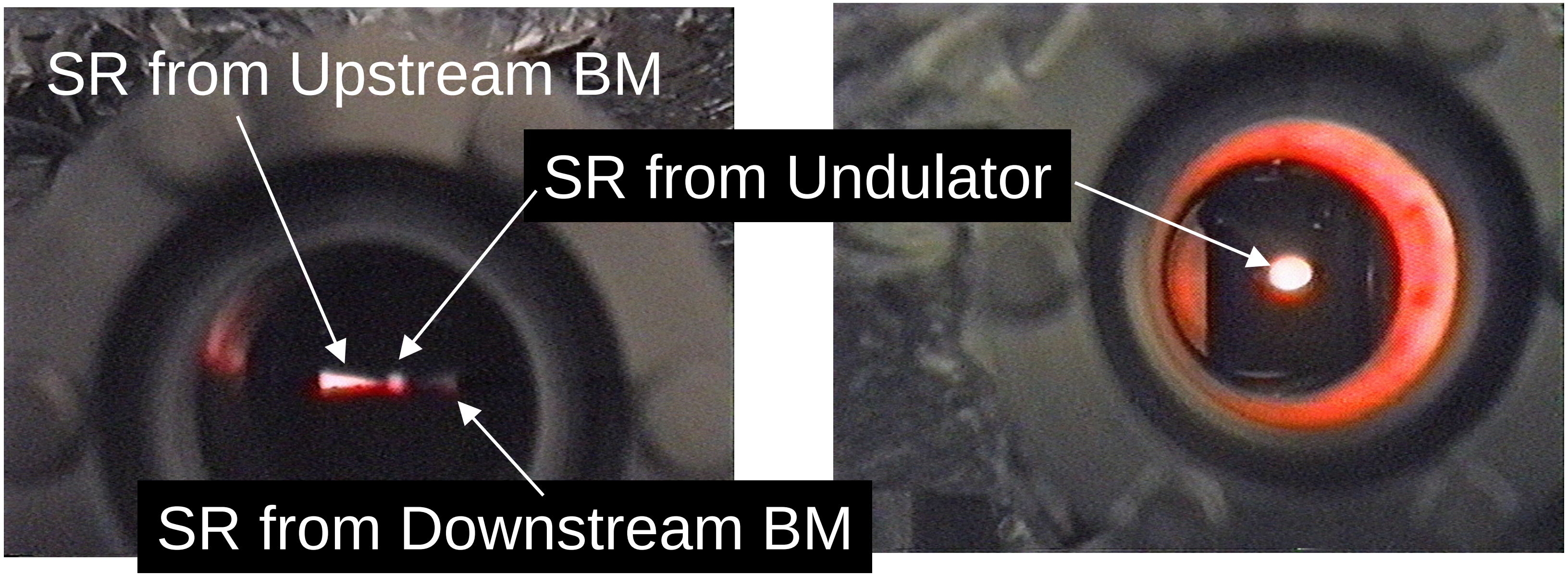
Example of ID Magnets

Halbach-type Magnet Array for SPring-8 Standard Undulators



Example of SR Image

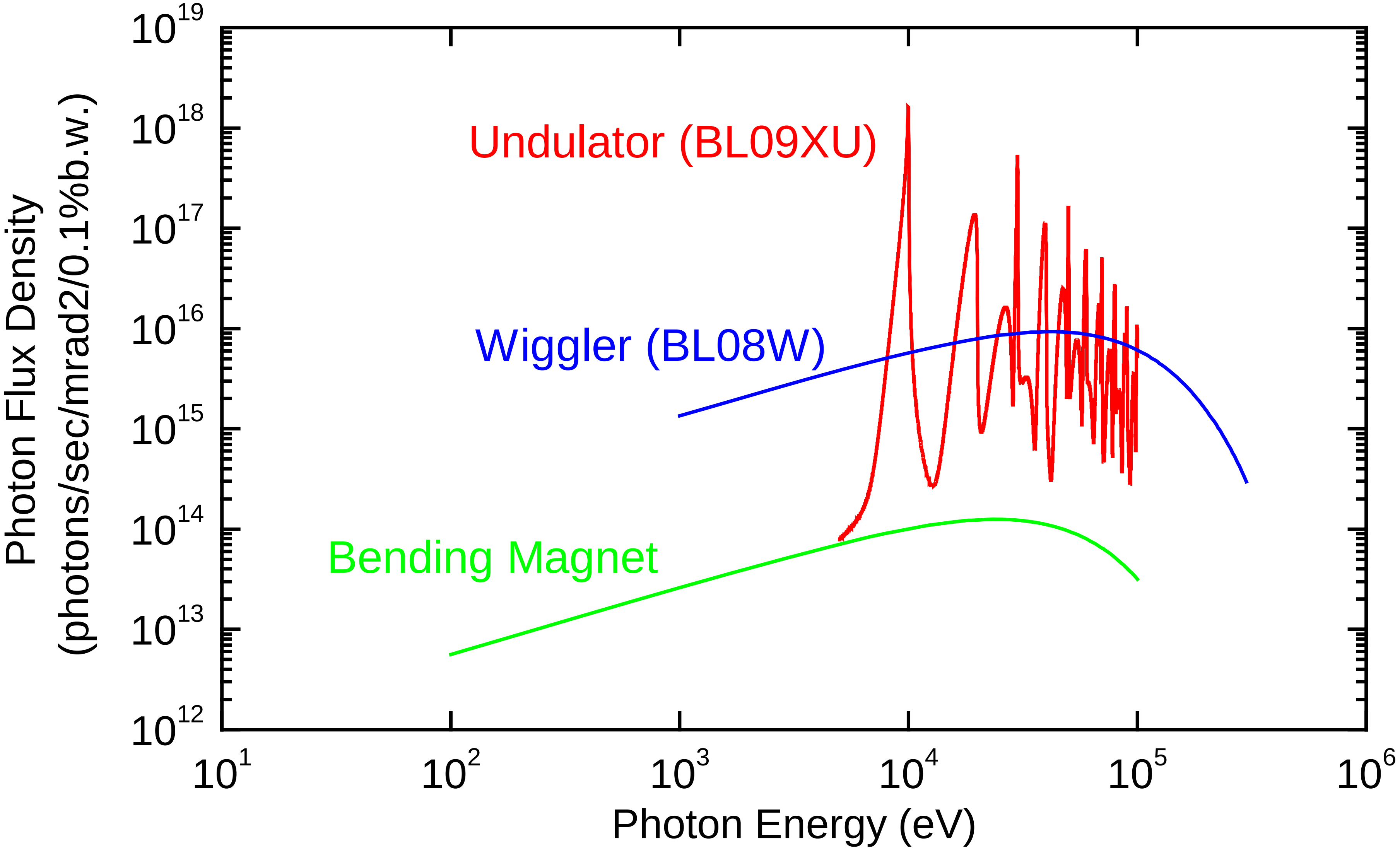
BL41XU@SP-8, First Image of SR
at Fluorescent Screen (<0.1mA)



Undulator Gap = 50 mm

Undulator Gap = 20 mm

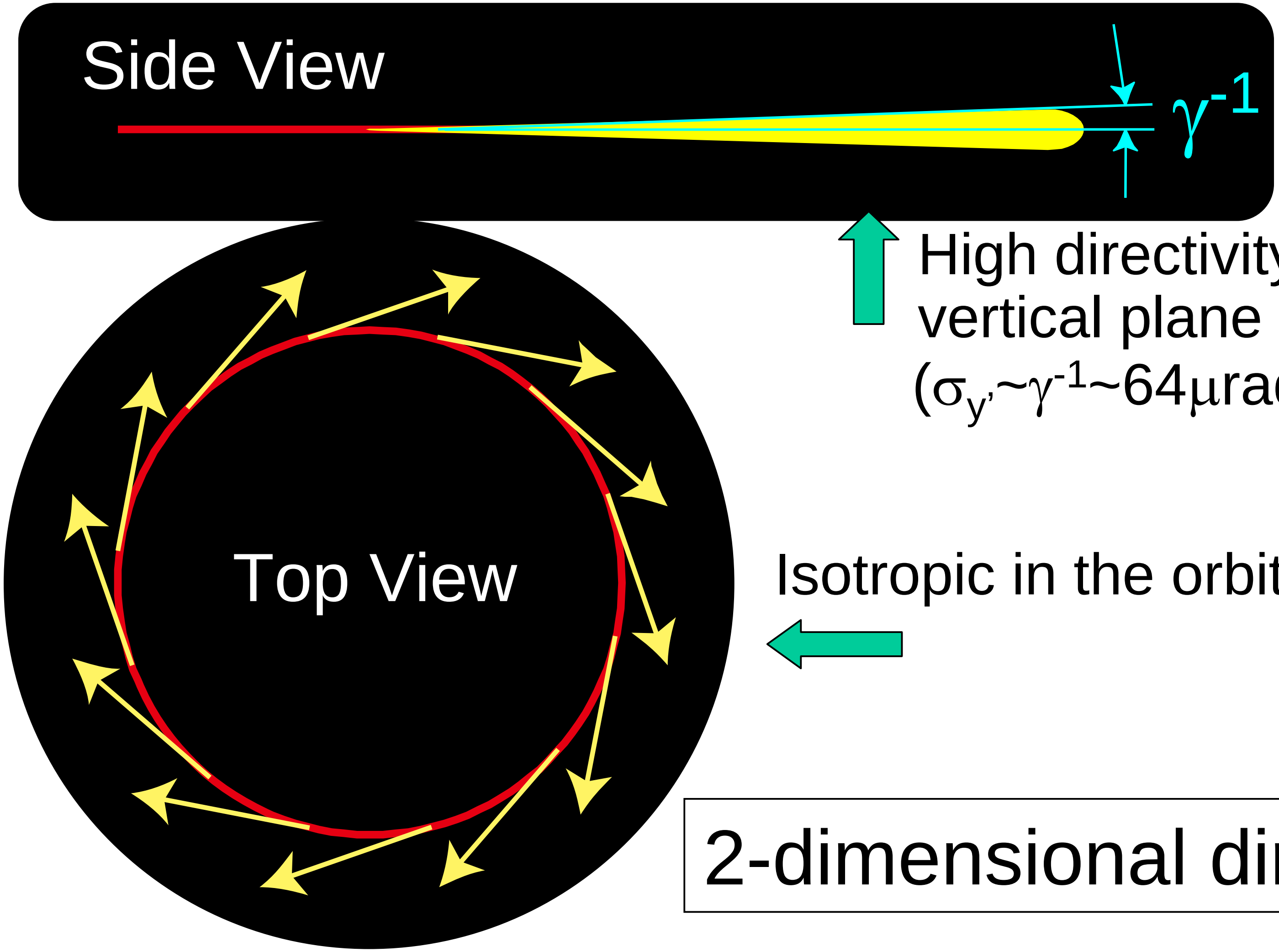
Comparison of Light Sources



Characteristics of SR (1)

- Radiation from BMs

Directivity of BM Radiation

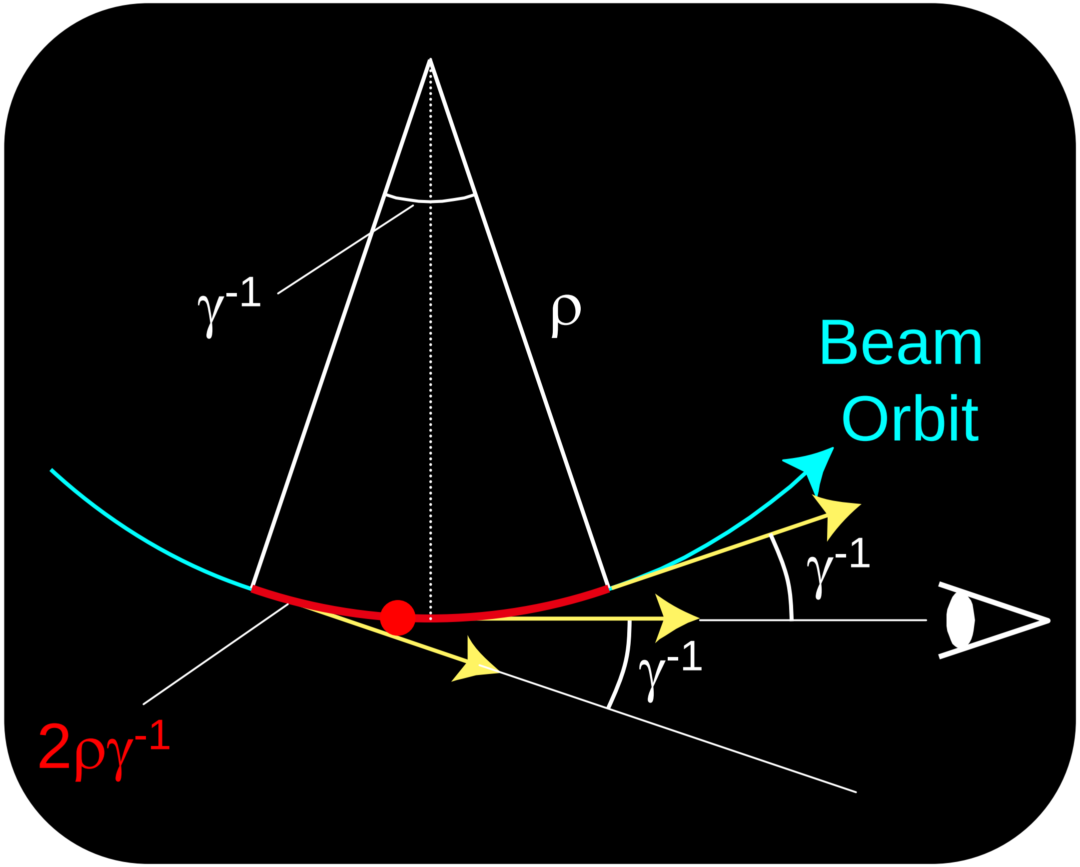


High directivity in the vertical plane ($\sigma_{y'} \sim \gamma^{-1} \sim 64\mu$rad@SP-8)

Isotropic in the orbital plane

2-dimensional directivity

Spectrum of BM Radiation (1)



Major contribution of radiation is from the portion painted red

Pulse duration for e^-

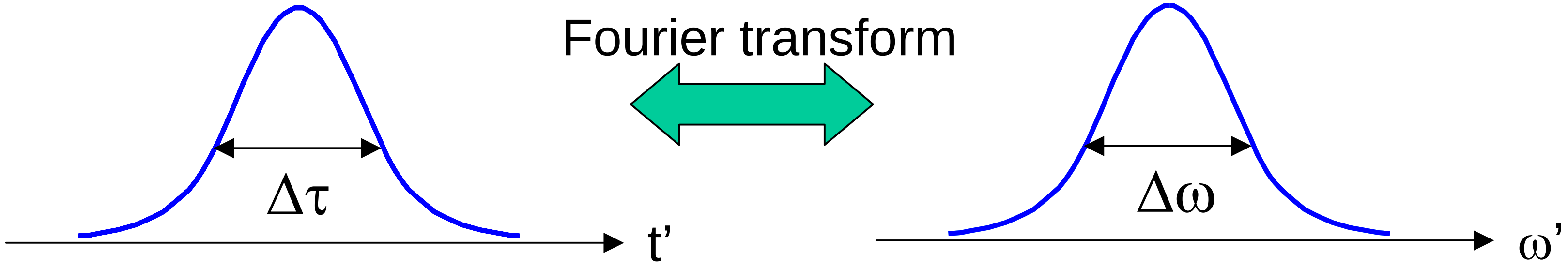
$$\Delta t = 2\rho\gamma^{-1}/c$$

squeezing

Pulse duration for observer

$$\Delta\tau = \frac{\Delta t}{2\gamma^2} = \frac{\rho}{\gamma^3 c}$$

Spectrum of BM Radiation (2)



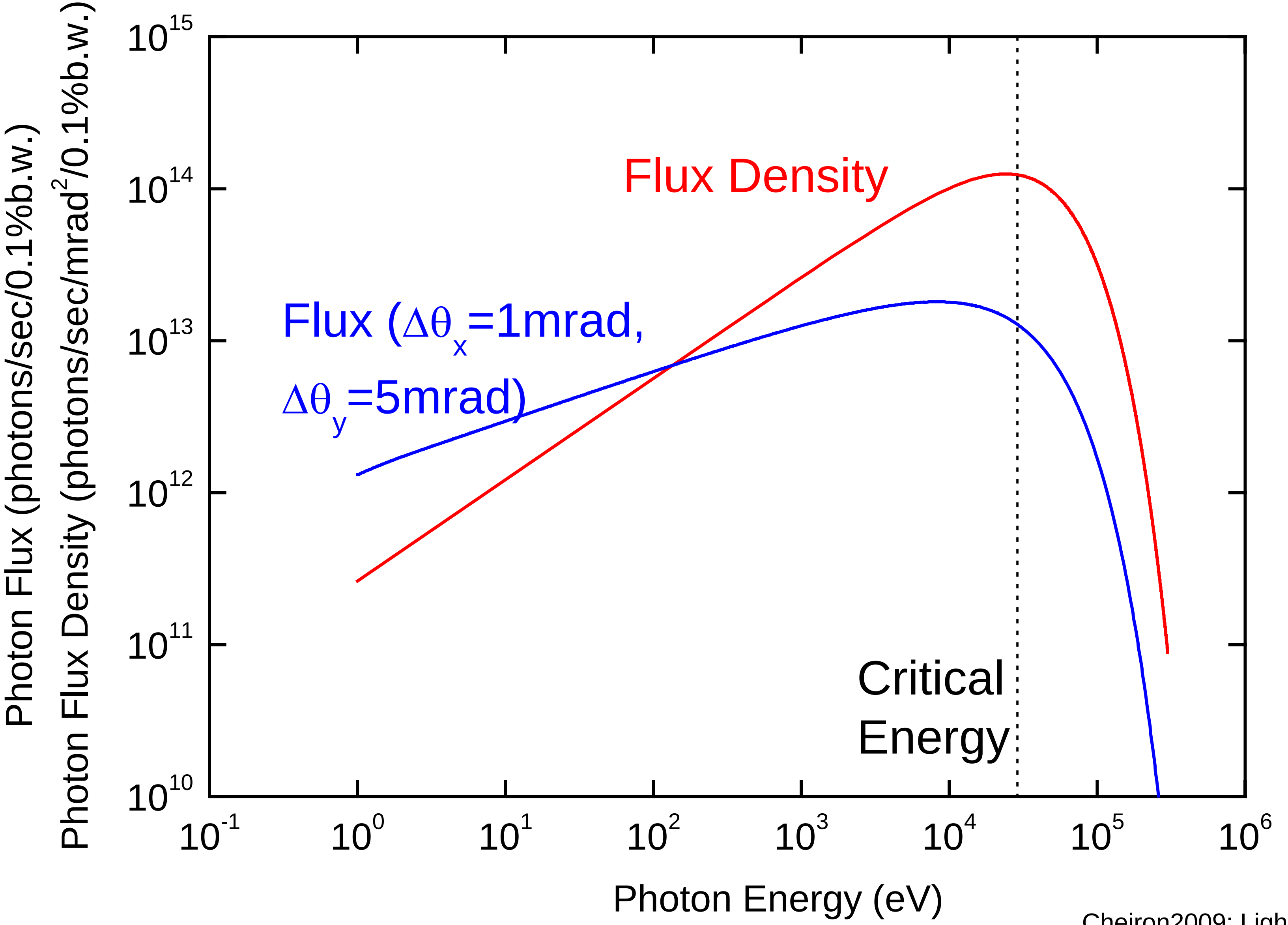
Light pulse with duration of $\Delta\tau$

White spectrum up to $\omega \sim \Delta\omega \sim 1/\Delta\tau$

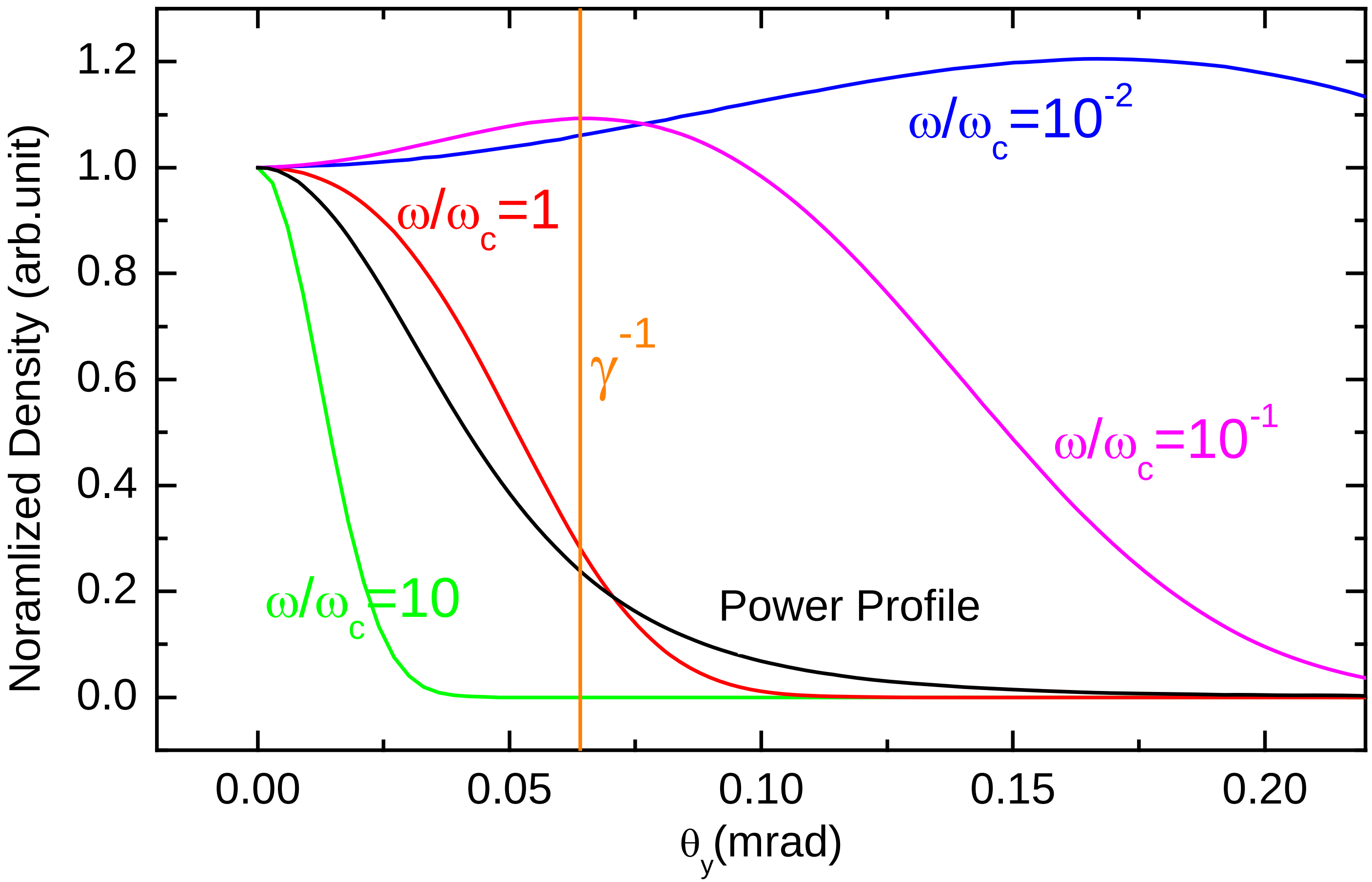
- By definition, $\omega_c = (3/2)\Delta\tau = 3\gamma^3 c/2\rho$ is called “critical frequency” of SR, which gives a criterion of the maximum energy of SR from a BM.
- In practical units,

$$\hbar\omega_c(\text{keV}) = 0.665 E_e^2(\text{GeV}) B(\text{T})$$

Example of Spectrum



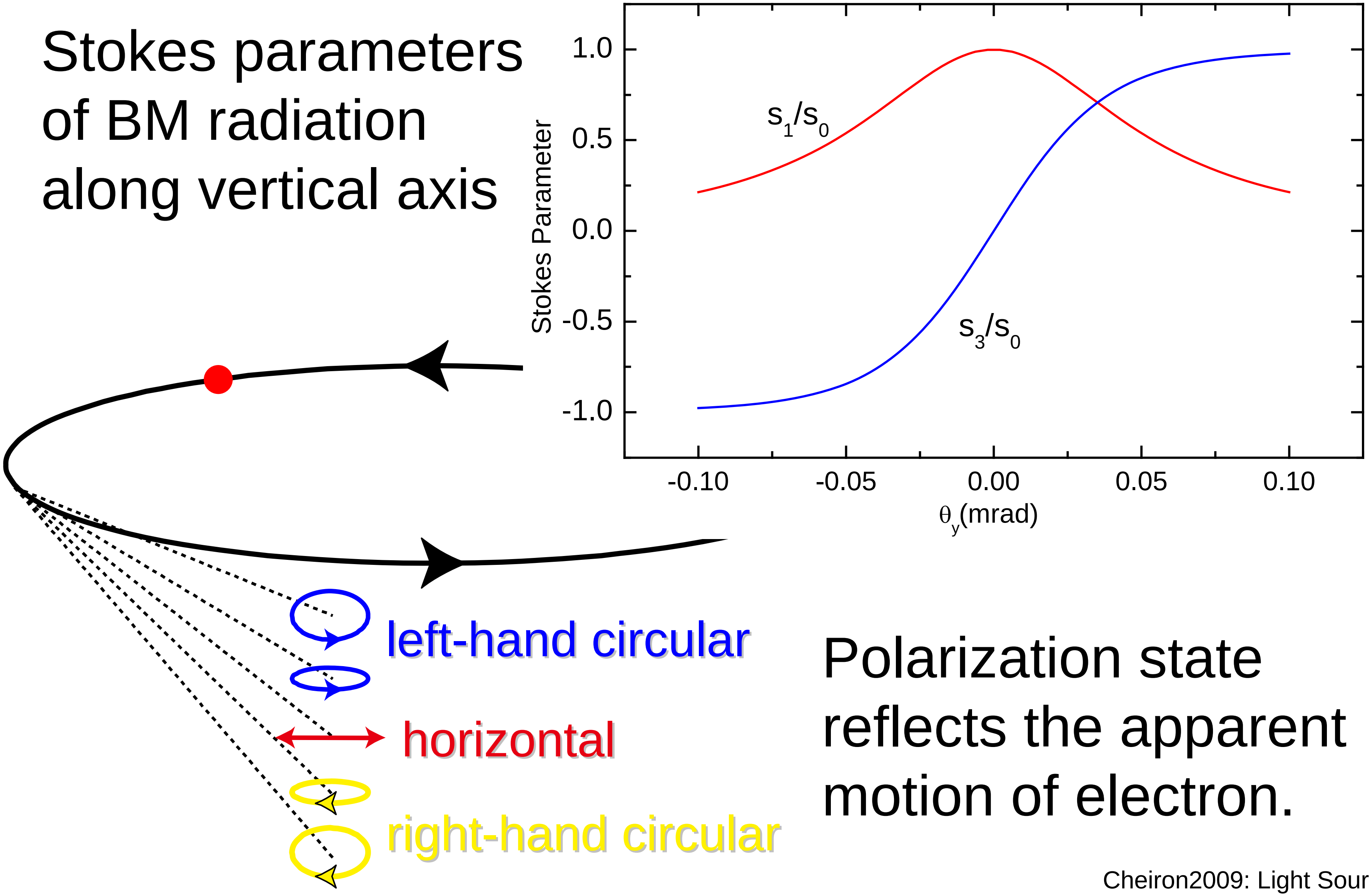
Angular Profile of BM Radiation



- power profile ~ flux profile@ω/ω_c=1
- larger angular divergence for lower energy

Polarization of BM Radiation

Stokes parameters of BM radiation along vertical axis



Polarization state reflects the apparent motion of electron.